

Swiss Finance Institute  
Research Paper Series N°13-20

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# On the strategic value of risk management<sup>1</sup>

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Revised Version, March 26 2013

<sup>1</sup>We thank Patrick Rey, Bill Rogerson, Jean Tirole, and participants to the finance and economics seminars at HEC Montréal and Paris-Dauphine University, and to the IDEI/Northwestern 2010 annual conference for their insightful comments. We are also grateful to the editor, Daniel Spulber, and two anonymous referees for carefully reviewing a previous version and providing very useful suggestions. The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) grant agreement 249415-RMAC, and from NCCR FinRisk (project on Banking and Regulation).

## Abstract

This article examines how firms facing volatile input prices and holding some degree of market power in their product market link their risk management with their production or pricing strategies. This issue is relevant in many industries ranging from manufacturing to energy retailing, where risk averse firms decide on their hedging strategies *before* their product market strategies. We find that hedging modifies the pricing and production strategies of firms. This strategic effect is channelled through the risk-adjusted expected cost, i.e., the expected marginal cost under the measure induced by shareholders' risk aversion. It has diametrically opposed impacts depending on the nature of product market competition: hedging toughens quantity competition while it softens price competition. Finally, committing to a hedging strategy is always a best response to non committing, and is a dominant strategy if firms compete à la Hotelling.

**Keywords:** Risk management, Imperfect Competition

**JEL Classification:** L13, G32

## 1 Introduction

Most formal analyses of corporate risk management decisions (for example Froot and Stein (1998), Rochet and Villeneuve (2011), and Bolton, Chen and Wang (2011)) describe a single price-taking firm that faces volatile cash flows and optimizes its mix of reserves (cash or debt capacity) and derivatives instruments (forward and options).

In many instances, this analysis constitutes a valid representation of reality. Firms producing commodities and raw materials (e.g., metals and minerals, oil and gas, electric power) face output price volatility that translates directly into cash flow volatility. In many industries, such as manufacturing, food processing, transportation, and energy retailing, firms face input price, not output price, volatility. When firms have no market power in their product market, the single-firm risk management logic applies. Individual hedging demand from these price-taking firms can then be aggregated to determine the equilibrium price of risk (for example, Bessembiner and Lemmon (2002), and Aid et al. (2011)).

If, however, firms facing input price volatility have some degree of market power in their product market, their strategies become more elaborate. For example, a firm can pass through to customers a portion of the input cost increase and/or can retain a portion of the input cost decrease. However, by modifying product price, the firm alters the competitive dynamics in its industry. It must therefore take into account the behavior of other firms, and the pass-through is determined in equilibrium.

The British electric power retailing sector provides a clear example of the strategic aspects of hedging. The British regulator (Ofgem (2008), page 10) indicates that: "there is evidence that the (6 largest suppliers) seek to benchmark their hedging strategies against each other in order to minimize the risk of their wholesale costs diverging materially from the competition". Suppliers then play a symmetric Nash equilibrium. What matters to them is not the absolute value of their hedging position, but its value relative to their competitors.

This article examines how hedging interacts with product market strategy when firms compete in quantity (Cournot) and in price (differentiated Bertrand).

There is a small academic literature on the strategic aspects of hedging. Closest to this work, Allaz and Villa (1993) examine the interplay of forward

and spot markets, and find that the availability of forward contracts reduces firms' market power in the spot market. However, their analysis differs from ours in a critical aspect: in their setting, firms sell their output on the forward or spot markets, where they exert market power, while in ours, firms exert no market power in the spot and forward markets for input. Adam, Dasgupta, and Titman (2007) examine two-period games in the presence of financial constraints: firms' hedging decision in the first-period affects their investment capacity, hence their profitability in the second period. They show that asymmetric equilibria arise: in equilibrium, some firms hedge, while others do not. In their model, the presence of financial constraints and the resulting potential underinvestment is the conduit for strategic interaction. Similarly, Loss (2012) examines the interaction between hedging demand and the characteristics of investment opportunities in the presence of financial constraints. He finds that a firm's hedging demand is high when investments are strategic substitutes, and low when they are strategic complements. In this article, by contrast, pricing and hedging are part of the same strategy. Bodnar, Dumas, and Marston (2002) consider a duopoly with asymmetric exposure to an exchange rate, and determine the optimal pass-through and related exposure. While the problem is related to the one examined here, the analytical approach is very different: they treat the exchange rate as a fixed input price, not as a stochastic variable. This article's contribution is therefore to analyze the strategic interactions between product market and hedging decisions of large corporations.

We focus the analysis on risk-averse firms that hedge before deciding their product market strategies. The empirical relevance of this choice is justified in Section 2. Formally, we use two-stage games: firms first determine their hedging strategy, then determine their product market strategy (quantity or price), conditional on their first stage choice.

We first analyze quantity competition. The necessary first-order conditions characterizing the equilibrium of the production game are similar to the standard Cournot case, except that risk-adjusted expected costs replace marginal costs. Investors value a marginal cost increase using the probability measure induced by their marginal utility of wealth in each state of the world, and not the physical probability measure. This risk-adjusted expected marginal cost is determined in equilibrium. It is decreasing in own hedging, and increasing in own production at the equilibrium. Thus, if a firm increases its hedging, it becomes more aggressive (Lemma 2).

An equilibrium of the production game always exists. If the variation in firms' absolute risk aversion remains small (this statement is made precise in Section 5), this equilibrium is unique, and an increase in own hedging reduces the other firm's equilibrium output (Proposition 1). If a symmetric equilibrium of the hedging game exists, and the variation in firms' absolute risk aversion remains small, hedging *toughens* quantity competition: firms hedge more than their (anticipated) equilibrium production, thus commit themselves to produce more than if their costs were constant and equal to the expected cost under the physical probability measure (Proposition 2).

We establish similar results for differentiated price competition, although with diametrically opposed implications. Risk-adjusted expected marginal costs, determined in equilibrium, replace constant marginal costs in the first-order conditions characterizing the equilibrium of the pricing game. They are decreasing in own hedging and decreasing in own price at the equilibrium. Thus, if a firm increases its hedging, it becomes more aggressive (Lemma 3).

An equilibrium of the pricing game always exists. If absolute risk aversion is constant, the equilibrium is unique, and an increase in own hedging reduces the other firm's equilibrium price. The crucial difference with quantity competition is that, if a symmetric equilibrium of the hedging game exists and absolute risk aversion is constant, hedging *softens* quantity competition: firms hedge less than their (anticipated) equilibrium production, thus commit themselves to a price higher than if their cost was constant and equal to the expected cost under the physical probability measure (Proposition 3).

For ease of exposition, the unicity and comparative statics results when firms compete in quantity are derived under the strong condition that risk aversion is constant. A weaker sufficient condition is that the variation of firms' absolute risk aversion remains small (Proposition 4).

Finally, we examine the strategic incentives to commit to a hedging position (Proposition 5). So far, we have assumed that Boards of Directors impose that firms commit to their hedging position. This is usually meant to limit speculation by traders. Ignoring that objective, does commitment arise in equilibrium? We first prove that committing is a firm's best response to the other not committing. This result is all the more striking that both firms committing is Pareto dominated if firms compete in quantity. On the other hand, if firms compete in price, commitment to a hedging strategy softens price competition. If total demand is inelastic (Hotelling competition)

and absolute risk aversion constant, the expected gain in profits more than compensates the volatility increase, and commitment is a dominant strategy. Thus, whether firms compete in quantity or in price, they have a strategic incentive to commit to their hedging decisions.

This article is structured as follows: Section 2 presents the model. Section 3 analyzes quantity competition. Section 4 analyzes price competition. Section 5 discusses robustness of the results. Section 6 examines incentives to commit to hedging decisions. Finally, Section 7 discusses further research. Technical proofs are presented in the Appendix.

## 2 The model

### 2.1 Demand and commercial profits

Consider two identical firms, indexed by  $i = 1, 2$ , competing à la Cournot. Firm  $i$  produces output  $q_i$ , total production is  $Q = q_1 + q_2$ , and inverse demand  $P(Q)$ . To produce one unit of good, both firms use one unit of the same input, at cost  $\tilde{c}$ . Firm  $i$ 's commercial profits for input cost  $\tilde{c}$  is:

$$\pi^C(q_i, q_j; \tilde{c}) = q_i (P(Q) - \tilde{c}).$$

**Assumption 1** *For all  $Q \geq 0$ , the inverse demand function  $P(\cdot)$  satisfies*

$$\frac{QP''(Q)}{(-P'(Q))} < 1 \tag{1}$$

and

$$\lim_{Q \rightarrow \infty} A(Q) = 0 \text{ and } \lim_{Q \rightarrow 0} A(Q) = \lim_{Q \rightarrow 0} P(Q) = +\infty$$

where

$$A(Q) = 2P(Q) + QP'(Q).$$

### 2.2 Deterministic input cost

Before introducing uncertainty, it is useful to briefly review the properties of the equilibrium when input costs are non-random and equal to  $c_i$  for each firm  $i$ . Firm  $i$ 's profit is:

$$\pi^C(q_i, q_j; c_i) = q_i (P(Q) - c_i).$$

When no confusion is possible, we use the following – shorter – notation:  $f_i = f(x_i, x_j)$ , i.e.,  $f_i$  is the value of function  $f(\cdot, \cdot)$  for firm  $i$  that plays  $x_i$



while firm  $j$  plays  $x_j$ . For example,  $\pi_i^C = \pi^C(q_i, q_j; \tilde{c})$ . Then,

$$\frac{\partial \pi_i^C}{\partial q_i} = (P(Q) - c_i) + q_i P'(Q) \text{ and } \frac{\partial^2 \pi_i^C}{(\partial q_i)^2} = 2P'(Q) + q_i P''(Q).$$

**Lemma 1** *Assumption 1 implies that, for all  $(c_1, c_2)$  there exists a unique Cournot equilibrium  $(q^E(c_1, c_2), q^E(c_2, c_1))$ . When this equilibrium is interior ( $q_i^E = q^E(c_i, c_j) > 0$  for  $i = 1, 2$ ), it satisfies*

$$\frac{\partial q_i^E}{\partial c_i} < 0 \text{ and } \frac{\partial q_j^E}{\partial c_i} > 0$$

for  $i = 1, 2$ .

**Proof.** Condition 1 guarantees that  $\frac{\partial^2 \pi_i^C}{(\partial q_i)^2} < 0$ . Thus, if a Cournot equilibrium exists, it is characterized by the necessary first-order conditions:

$$\frac{\partial \pi_i^C}{\partial q_i} = (P(Q) - c_i) + q_i P'(Q) = 0. \quad (2)$$

Assumption 1 guarantees that, for all  $c > 0$ , the equation

$$A(Q) = 2P(Q) + QP'(Q) = c$$

admits a unique solution  $Q^E(c)$ . When the equilibrium is interior ( $q_i^E > 0$  for  $i = 1, 2$ ), the equilibrium quantities are:

$$q^E(c_i, c_j) = \frac{P(Q^E(c_i + c_j)) - c_i}{(-P'(Q^E(c_i + c_j)))}.$$

Finally, we verify that:

$$\frac{\partial q_i^E}{\partial c_i} = \frac{\partial q^E(c_i, c_j)}{\partial c_i} = \frac{2P'(Q^E) + q_j^E P''(Q^E)}{P'(Q^E) (3P'(Q^E) + Q^E P''(Q^E))} < 0 \quad (3)$$

and

$$\frac{\partial q_j^E}{\partial c_i} = \frac{\partial q^E(c_j, c_i)}{\partial c_i} = -\frac{P'(Q^E) + q_i^E P''(Q^E)}{P'(Q^E) (3P'(Q^E) + Q^E P''(Q^E))} > 0. \quad (4)$$

■

Thus, an increase in firm  $i$ 's marginal cost induces a reduction in its Cournot equilibrium output  $q_i^E$  and an increase in the equilibrium output

of its competitor  $q_j^E$ .

### 2.3 Uncertainty on input costs and risk management

We return to the random input cost case. Ex ante, the input cost  $\tilde{c}$  is a random variable, distributed following cumulative distribution function  $G(\cdot)$  with bounded support  $\mathfrak{C} = [\underline{c}, \bar{c}]$ .  $\mathfrak{C}$  and  $G(\cdot)$  are common knowledge to both firms.

Firms can purchase input in the wholesale forward market at (forward) price  $F$ . To eliminate speculative motives for hedging, we assume that  $F = \mathbb{E}[\tilde{c}]$ . There are no transaction costs associated with hedging.

Firms do not exert market power in the spot and forward wholesale markets for input, even though they do exert market power in their product market. For example, airlines do not exert market power in the fuel market, yet they are an oligopoly on specific routes (see for example Gerardi and Shapiro (2009)); food processing firms may not exert market power in the feedstock market, while most empirical studies document market power in their product markets (see for example Sheldon and Sperling's (2001) survey).

We also assume that firms choose their outputs (or prices) before input costs are realized. This is the case for most manufacturing industries<sup>1</sup>: producers (e.g., car manufacturers) commit to a product price or volume for the relevant period (typically one or two quarters). During that period, input prices (e.g., aluminum and steel prices) vary. This assumption also holds for the electricity supply industry in the United Kingdom, where retail rates typically change only 3 or 4 times a year, while wholesale power prices vary continuously. This price inflexibility could be due for example to the high cost of adjusting prices, or simply to industry practices.

Two other assumptions are crucial for risk management to have any strategic value. First, each firm must be able to publicly commit to its hedging decision. Second, firms must be "risk averse", in the sense that shareholder value can be measured by the expectation of some concave function of profit. We now motivate these assumptions.

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<sup>1</sup>This assumption does not apply to industries where output price is flexible. For example delivery services (e.g., Fedex and UPS) explicitly include in their published rate a fuel surcharge schedule, that depends on the price of an oil index. Similarly, electricity suppliers in Norway offer retail contracts explicitly adjusted to the wholesale power price.

When firms set production after the input price is realized, the profit from the hedge is known before the production decision is made, thus has no impact on it. Firms cannot do any better than standard deterministic profit maximization. Knowing that, when firms make the hedging decision, they follow the "standard" one-firm risk management logic.

## 2.4 Public commitment to hedging decision

We assume throughout this article that firms can publicly commit to their hedging decisions  $(H_1, H_2)$  before they select their output (or prices). This assumption can be justified as follows.

First, financial regulations require firms to publish in their quarterly statements a description of their portfolio of forward purchases and sales. While some discretion still exists in disclosure, an outside party can get a close picture of a firm's hedging portfolio. For example, Jin and Jorion (2007) were able to compute the delta-equivalent of the forward portfolio for US oil and gas companies. Also, as previously mentioned, electricity suppliers in Britain infer each other's hedging portfolio from financial statements and other public information.

Second, industrial firms can – and in practice do – commit to a hedging strategy through their risk management policy. Forward sales and purchases, that require the use of derivatives, are usually handled with extreme caution by Board of Directors, concerned about potential speculative behavior by traders. Boards then require management to define and follow a clear hedging strategy, often declining in time: for example "hedge fully our exposure for the next quarter, hedge half of the exposure for the next two quarters, and a fourth of our exposure for the quarter after". As mentioned earlier, this strategy is communicated to investors and regulators. Management has then limited discretion to deviate from this strategy.

Without commitment to hedging, no strategic interaction arises, i.e., there exists a dichotomy between hedging and production. Suppose indeed that firms select output, then hedge. Reasoning backwards, consider first the hedging decision, once production is known. Since (i) firms are risk averse, and (ii) there are no transaction costs nor expected gain from hedging (i.e.,  $\mathbb{E}[\tilde{c}] - F = 0$ ), full hedging is the optimal strategy. Consider now the production decision. Knowing that input costs will be perfectly covered at the forward price, firms play a symmetric Cournot game with constant marginal costs equal to  $F$ , thus their equilibrium output is  $q_1^E = q_2^E = q^E(F, F)$ .

Thus, this article is focussed on situations where risk-averse firms hedge before making their production decision.

## 2.5 Objective function

At  $t = 2$  (i.e., once the input cost  $\tilde{c}$  is known), the profit function of firm  $i$  that has purchased forward quantity  $H_i$  at the forward price  $F$  is:

$$\begin{aligned}\pi_i &= \pi(q_i, q_j, H_i; \tilde{c}) = q_i(P(Q) - \tilde{c}) + H_i(\tilde{c} - F) \\ &= q_i(P(Q) - F) + (H_i - q_i)(\tilde{c} - F).\end{aligned}$$

The first expression of firm  $i$ 's profits reflects a purchase of input volume  $q_i$  at cost  $\tilde{c}$ , and profit  $(\tilde{c} - F)$  on each of the  $H_i$  units purchased forward at  $t = 0$ . Alternatively, the firm can consider that its production cost is  $F$ , and its exposure to input price fluctuations is  $(H_i - q_i)$ .

To obtain a strategic impact of risk management, a crucial ingredient is that firms be risk-averse. Thus, we assume that firms maximize some expected utility of profits  $U(\pi_i)$  where  $U(\cdot)$  is increasing and (weakly) concave. There are several possible justifications for this assumption. In the case of a small firm, owned and managed by the same person, the objective of this owner-manager is to maximize the expected utility of her wealth, the profit of the firm being a large component of this wealth.

However, we are mostly interested in large firms, typically owned by a diffuse population of small shareholders, and whose managers own only a small fraction of the shares. If managers act in the best interest of diversified shareholders and the stock market is frictionless, the objective should be to maximize the expected present value of future profits, where the expectation is taken under the risk adjusted distribution, which incorporates risk premia. Such a framework is consistent with the Modigliani and Miller theorem, and implies in particular that corporate risk management does not create any value for shareholders.

A more realistic description of reality incorporates financial frictions, typically in the form of a wedge between the costs of external and internal finance (Froot, Scharfstein, and Stein (1993)), transaction costs of primary security markets (Décamps et al. (2011)), or agency costs (DeMarzo and Sannikov (2006), Biais et al. (2007)). In each of cases, shareholder value (once firms optimize their financing and investment policies) can be represented as the expectation of a concave function of future profits. Even when shareholders are risk neutral (or completely diversified), financial frictions generate risk aversion in the (indirect) preferences of shareholders.

This is why we represent shareholders' preferences by a concave function of future profits, denoted  $U(\pi)$ . For simplicity, we consider a symmetric model where  $U(\cdot)$  is the same for all firms. We do not attempt to endogenize  $U(\cdot)$  by modelling explicitly the financial frictions and the optimal financing and investment policies that give rise to this function  $U(\cdot)$ .

Consistent with the literature reviewed above, we further assume that absolute risk aversion  $\rho(\pi) = \left(-\frac{U''}{U'}\right)(\pi)$  is non-increasing.

At date  $t = 0$ , the shareholder value of firm  $i$  is then:

$$\begin{aligned} v_i &= v(q_i, q_j, H_i) = \mathbb{E}[U(\pi(q_i, q_j, H_i; \tilde{c}))] \\ &= \mathbb{E}[U(q_i(P(Q) - F) + (H_i - q_i)(\tilde{c} - F))] \end{aligned}$$

We look for subgame perfect equilibria of the two-stage games played by firms. We solve by backward induction: we first determine the unique second-stage equilibrium  $(q^*(H_1, H_2), q^*(H_2, H_1))$ , then insert its value into  $v_i$  to obtain the first-stage payoff functions, denoted  $V_i = V(H_i, H_j)$ .

### 3 Strategic use of hedging

#### 3.1 Random input costs: an illustrative example

Before solving the general case, we illustrate the main insights using a simple example: (i) absolute risk aversion is constant and equal to  $\rho$ , which corresponds to  $U(x) = 1 - \exp(-\rho x)$ , (ii) inverse demand is linear  $P(Q) = 1 - Q$ , and (iii) input cost  $\tilde{c}$  is normally distributed<sup>2</sup>, with mean  $F$  and standard deviation  $\sigma$ . In this case,

$$\begin{aligned} v(q_i, q_j, H_i) &= \mathbb{E}[1 - \exp(-\rho(q_i(P(Q) - F) + (H_i - q_i)(\tilde{c} - F)))] \\ &= 1 - \exp(-\rho m(q_i, q_j, H_i)) \end{aligned}$$

where

$$m_i = m(q_i, q_j, H_i) = q_i(P(Q) - F) - \frac{\rho(H_i - q_i)^2 \sigma^2}{2}$$

is the certainty equivalent of firm  $i$ 's profit. Maximizing  $v_i$  is equivalent to maximizing  $m_i$ . Now,

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<sup>2</sup>The support of the distribution of cost is not bounded. However, as we will prove below, the main results still hold.

$$\frac{\partial m_i}{\partial q_i} = P(Q) - F + q_i P'(Q) + \rho \sigma^2 (H_i - q_i),$$

and

$$\frac{\partial^2 m_i}{(\partial q_i)^2} = 2P'(Q) + q_i P''(Q) - \rho \sigma^2 < 0.$$

Replacing  $P(Q)$  by its expression, the first-order necessary and sufficient conditions characterizing the equilibrium are

$$q_i (2 + \rho \sigma^2) + q_j = 1 - F + \rho \sigma^2 H_i \quad (5)$$

for  $i = 1, 2$ , which yield the unique equilibrium for the second-stage game:

$$q_i^* = q^*(H_i, H_j) = \frac{1}{3 + \rho \sigma^2} \left( 1 - F + \frac{\rho \sigma^2}{1 + \rho \sigma^2} ((2 + \rho \sigma^2) H_i - H_j) \right)$$

for  $i = 1, 2$ . Thus,  $V(H_i, H_j) = 1 - \exp(-\rho M(H_i, H_j))$ , where

$$M_i = M(H_i, H_j) = m(q^*(H_i, H_j), q^*(H_j, H_i), H_i)$$

is the certainty equivalent of firm  $i$ 's profit in the production game (stage 2).

Now,

$$\begin{aligned} \frac{\partial M_i}{\partial H_i} &= \frac{\partial m_i}{\partial q_i} \frac{\partial q_i^*}{\partial H_i} + \frac{\partial m_i}{\partial q_j} \frac{\partial q_j^*}{\partial H_i} + \frac{\partial m_i}{\partial H_i} \\ &= \left( q_i^* P'(Q) \frac{\partial q_j^*}{\partial H_i} - (H_i - q_i^*) \rho \sigma^2 \right) \\ &= -\rho \sigma^2 \left( H_i - \left( 1 + \frac{1}{(3 + \rho \sigma^2)(1 + \rho \sigma^2)} \right) q_i^* \right). \end{aligned}$$

The necessary first-order conditions of the hedging game (stage 1) are then:

$$H_i^* = \left( 1 + \frac{1}{(3 + \rho \sigma^2)(1 + \rho \sigma^2)} \right) q_i^*.$$

Since  $M$  is concave in its first argument, these conditions are also sufficient.

Replacing  $q_i^*$  by  $q^*(H_i, H_j)$  and solving for the first period equilibrium yields a unique solution which is symmetric:

$$H^* = \frac{1 + \frac{1}{(3 + \rho \sigma^2)(1 + \rho \sigma^2)}}{1 - \frac{\rho \sigma^2}{3(3 + \rho \sigma^2)(1 + \rho \sigma^2)}} q^E(F, F),$$

where

$$q^E(F, F) = \frac{1 - F}{3}$$

is the unique symmetric equilibrium output if marginal cost is constant and equal to  $F$ . Equilibrium output is then:

$$q^* = \frac{1}{1 - \frac{\rho\sigma^2}{3(3+\rho\sigma^2)(1+\rho\sigma^2)}} q^E(F, F) > q^E(F, F).$$

Some key features of the analysis are worth noting. First,  $v(q_i, q_j, H_i)$  is concave in  $q_i$ , due to the concavity of the profit function  $\pi_i^C$  and to risk aversion.

Second, the first order conditions (5) of the production game can be rewritten as

$$2q_i + q_j = 1 - \hat{c}(q_i, H_i)$$

where

$$\hat{c}(q_i, H_i) \equiv \frac{\mathbb{E}[U'(\pi_i) \tilde{c}]}{\mathbb{E}[U'(\pi_i)]} = F + \rho\sigma^2(q_i - H_i)$$

is the risk-adjusted expected marginal cost of firm  $i$ . It is increasing in  $q_i$ , decreasing in  $H_i$ , and lower than the expected cost  $F$  if and only if  $H_i > q_i$ . Equilibrium of the production game is the solution of the system:

$$\begin{cases} q_1 = q^E(\hat{c}(q_1, H_1), \hat{c}(q_2, H_2)) \\ q_2 = q^E(\hat{c}(q_2, H_2), \hat{c}(q_1, H_1)) \end{cases}.$$

Finally, at the equilibrium, firms hedge more than they produce, i.e.,  $H^* > q^*$ , which yields a risk-adjusted cost lower than  $F$ . Thus, firms produce more than if costs were non random (and equal to  $F$ ):  $q^* > q^E(F, F)$ . Random input costs lead to higher output and lower equilibrium prices.

As we will see in the remainder of this article, these features also hold under more general conditions on utility, demand, and input cost distribution.

### 3.2 Equilibrium of the production game

Consider now a general specification.

$$\begin{aligned}\frac{\partial v_i}{\partial q_i} &= \mathbb{E} \left[ U'(\pi_i) \frac{\partial \pi_i}{\partial q_i} \right] = \mathbb{E} \left[ U'(\pi_i) \left( P(Q) - \tilde{c} + q_i P'(Q) \right) \right] \\ &= \mathbb{E} [U'(\pi_i)] \left( P(Q) + q_i P'(Q) \right) - \mathbb{E} [U'(\pi_i) \tilde{c}] \\ &= \mathbb{E} [U'(\pi_i)] \left( P(Q) + q_i P'(Q) - \hat{c}(q_i, q_j, H_i) \right)\end{aligned}$$

where

$$\hat{c}_i = \hat{c}(q_i, q_j, H_i) \equiv \frac{\mathbb{E} [U'(\pi(q_i, q_j, H_i; \tilde{c})) \tilde{c}]}{\mathbb{E} [U'(\pi(q_i, q_j, H_i; \tilde{c}))]} = F + \frac{\text{cov} [U'(\pi(q_i, q_j, H_i; \tilde{c})), \tilde{c}]}{\mathbb{E} [U'(\pi(q_i, q_j, H_i; \tilde{c}))]} \quad (6)$$

is the risk-adjusted expected cost of firm  $i$ .  $v(q_i, q_j, H_i)$  is concave in  $q_i$  since

$$\frac{\partial^2 v_i}{(\partial q_i)^2} = \mathbb{E} \left[ U''(\pi_i) \left( \frac{\partial \pi_i}{\partial q_i} \right)^2 + U'(\pi_i) \frac{\partial^2 \pi_i}{(\partial q_i)^2} \right] < 0.$$

Therefore, if an interior Nash equilibrium  $(q_i^*, q_j^*)$  exists, it is determined by the first order conditions

$$P(Q^*) + q_i^* P'(Q^*) - \hat{c}(q_i^*, q_j^*, H_i) = 0 \quad (7)$$

for  $i = 1, 2$ .

Before proving existence of a Nash equilibrium and deriving sufficient conditions for unicity, we examine equation (7). The interaction between hedging and production is channelled through the expected risk-adjusted cost  $\hat{c}(q_i, q_j, H_i)$ , determined in equilibrium. If the firm produces one more unit, it costs  $\tilde{c}$  in each state of the world. Shareholders value marginal cost using the probability measure induced by their marginal utility of wealth  $U'(\pi_i)$  in each state of the world, and not according to the physical probability measure. From the perspective of firm  $i$ 's shareholders, the risk-adjusted expectation of any random variable  $x(\tilde{c})$  is:

$$\hat{\mathbb{E}}_i [x(\tilde{c})] = \frac{\mathbb{E} [U'(\pi(q_i, q_j, H_i; \tilde{c})) \times x(\tilde{c})]}{\mathbb{E} [U'(\pi(q_i, q_j, H_i; \tilde{c}))]}.$$

The impact of  $q_i$ ,  $q_j$ , and  $H_i$  on the expected risk-adjusted cost is summarized in the following Lemma, proven in Appendix A.1:



**Lemma 2** For any  $(q_i, q_j, H_i)$ :

$$\frac{\partial \hat{c}_i}{\partial H_i} < 0 \text{ and } \hat{c}_i \leq F \Leftrightarrow H_i \geq q_i. \quad (8)$$

$$\frac{\partial \hat{c}_i}{\partial q_j} = -q_i P' (Q) \widehat{cov}_i [\rho (\pi_i), \tilde{c}], \quad (9)$$

thus:

$$\begin{cases} \frac{\partial \hat{c}_i}{\partial q_j} = 0 & \text{if } \rho (\pi_i) \text{ is constant} \\ \frac{\partial \hat{c}_i}{\partial q_j} < 0 \Leftrightarrow H_i > q_i & \text{if } \rho (\pi_i) \text{ is decreasing} \end{cases}. \quad (10)$$

Finally, for any  $(H_i, H_j)$ ,

$$\frac{\partial \hat{c}}{\partial q_i} (q_i^*, q_j^*, H_i) > 0. \quad (11)$$

A marginal increase in hedging increases marginal profit when  $\tilde{c} > F$ , and reduces it when  $\tilde{c} < F$ . However, favorable realizations are weighted by a lower marginal utility, hence, overall, the risk-adjusted expected cost decreases. Thus, "*ceteris paribus*", increasing  $H_i$  reduces firm  $i$  risk-adjusted expected marginal cost.

Dependency of  $\hat{c}_i$  with respect to  $q_i$  and  $q_j$  is indirect, channelled through the marginal utility of profits. If absolute risk-aversion is constant,  $\hat{c}_i$  depends only on  $q_i$  and  $H_i$ , which yields a more familiar Cournot game. Otherwise,  $q_j$ 's impact on  $\hat{c}_i$  depends on the sign of  $(H_i - q_i^*)$ . Finally, at the equilibrium output, expected risk-adjusted marginal cost is increasing.

We now turn to existence and unicity of the equilibrium of the production game. Since we ultimately focus on symmetric equilibria (where  $H_1^* = H_2^* = H$ ,  $q_1^* = q_2^* = q^*$ ,  $\hat{c}_1 = \hat{c}_2$ ), we restrict our attention to the case where  $|H_1 - H_2|$  is small enough that the equilibrium of the production game is interior.

$(q_1^*, q_2^*)$  is thus a fixed point of the function  $\Phi$ , defined from  $\mathbb{R}^2$  into  $\mathbb{R}^2$  by

$$\begin{cases} \Phi_1 (q_1, q_2) = q^E (\hat{c} (q_1, q_2, H_1), \hat{c} (q_2, q_1, H_2)) \\ \Phi_2 (q_1, q_2) = q^E (\hat{c} (q_2, q_1, H_2), \hat{c} (q_1, q_2, H_1)) \end{cases}.$$

**Proposition 1** For any  $(H_1, H_2)$  close enough to the diagonal, an equilibrium of the production game exists. If absolute risk aversion is constant, this equilibrium is unique, hence determined by  $q_i^* = q^* (H_i, H_j)$  for  $i = 1, 2$ . Furthermore, a marginal increase in firm  $i$ 's hedging reduces firm  $j$ 's equilibrium

output:

$$\frac{\partial q_j^*}{\partial H_i} < 0.$$

**Proof.** For existence, we apply Brouwer's fixed point theorem to the function  $\Phi$ . Since  $\tilde{c} \leq \bar{c}$  by assumption,

$$\hat{c}(q_i, q_j, H_i) = \frac{\mathbb{E}[U'(\pi(q_i, q_j, H_i; \tilde{c})) \times \tilde{c}]}{\mathbb{E}[U'(\pi(q_i, q_j, H_i; \tilde{c}))]} \leq \bar{c}$$

for all  $(q_i, q_j, H_i)$ . Thus, since  $\frac{\partial q^E}{\partial c_j}(c_i, c_j) \geq 0$  and  $\frac{\partial q^E}{\partial c_i}(c_i, c_j) \leq 0$ :

$$q^E(\hat{c}(q_i, q_j, H_i), \hat{c}(q_j, q_i, H_j)) \leq q^E(0, \bar{c}) \equiv \bar{q}^E.$$

Thus, we can limit our search to  $(q_i, q_j) \in [0, \bar{q}^E]^2$ . Since  $q^E(x, y)$  and  $\hat{c}(x, y, z)$  are continuous in all their arguments, and defined on a compact and convex set of  $\mathbb{R}^2$ , Brouwer theorem guarantees existence of an equilibrium.

If absolute risk aversion is constant, we prove in Appendix A.2 that the real part of the eigenvalues of the Jacobian matrix

$$J(q_1^*, q_2^*, H_1, H_2) = \begin{bmatrix} \frac{\partial q_1^E}{\partial q_1} - 1 & \frac{\partial q_1^E}{\partial q_2} \\ \frac{\partial q_2^E}{\partial q_1} & \frac{\partial q_2^E}{\partial q_2} - 1 \end{bmatrix}$$

are negative, which implies that the equilibrium is unique. Finally, constant risk aversion implies  $\frac{\partial \hat{c}}{\partial q_j}(q_i^*, q_j^*, H_i) = 0$  by Lemma 2. Firms play a familiar Cournot game with marginal costs  $\hat{c}_i$  increasing in  $q_i$  at the equilibrium, and decreasing in  $H_i$ . We prove in Appendix A.3 that, since increasing hedging reduces a firm's cost, it make her more aggressive, and reduces her competitor's output. ■

A marginal increase in  $H_i$  commits firm  $i$  to a higher output. This strategic effect can be understood through three equivalent intuitions. A first intuition is that a marginal increase in  $H_i$  reduces  $q_j^*$ , thus increases  $q_i^*$  (since quantities are strategic substitutes). A second intuition is that a marginal increase in  $H_i$  reduces the risk-adjusted cost, thus increases  $q_i^*$ . A final intuition is that a marginal increase in  $H_i$  increases the volume exposed to input price fluctuations  $(H_i - q_i)$ , thus firm  $i$  must increase  $q_i$  to reduce this exposure.

### 3.3 Equilibrium of the hedging game

Suppose a symmetric interior equilibrium of the hedging game  $(H^*, H^*)$  exists.

**Proposition 2** 1. A symmetric interior equilibrium is characterized by the necessary first-order conditions:

$$\left( \widehat{c}(q^*, q^*, H^*) - F + P'(Q^*) q^* \frac{\partial q_j^*}{\partial H_i} \right) (H^*, H^*) = 0 \quad (12)$$

for  $i = 1, 2$ , where  $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*)$  represents  $\frac{\partial q_j^*}{\partial H_i}(H_j, H_i) \Big|_{(H^*, H^*)}$ .

2. If absolute risk aversion is constant, hedging toughens quantity competition: through over-hedging ( $H^* > q^*$ ), firms commit to higher equilibrium output than if marginal costs were constant equal to  $F$

$$\widehat{c}(q^*, q^*, H^*) < F \text{ and } q^* > q^E(F, F).$$

**Proof.**

1. For  $i = 1, 2$ , the first-order conditions defining equilibrium hedging volume  $H_i^*$  are:

$$\mathbb{E} \left[ U'(\pi_i^*) \left( \frac{\partial \pi_i^*}{\partial H_i} + \frac{\partial \pi_i^*}{\partial q_j} \frac{\partial q_j^*}{\partial H_i} + \frac{\partial \pi_i^*}{\partial q_i} \frac{\partial q_i^*}{\partial H_i} \right) \right] = 0$$

where  $\pi_i^* = \pi(q_i^*, q_j^*, H_i^*)$ ,  $\frac{\partial \pi_i^*}{\partial x} = \frac{\partial \pi(q_i^*, q_j^*, H_i^*)}{\partial x}$  for  $x \in \{q_i, q_j, H_i\}$ ,  $\frac{\partial q_i^*}{\partial H_i} = \frac{\partial q_j^*}{\partial H_i}(H_i^*, H_j^*)$ , and  $\frac{\partial q_j^*}{\partial H_i} = \frac{\partial q_j^*}{\partial H_i}(H_j^*, H_i^*)$ . Since  $\frac{\partial \pi_i^*}{\partial H_i} = \tilde{c} - F$ ,  $\frac{\partial \pi_i^*}{\partial q_j} = P'(Q) q_i$ , and  $\frac{\partial \pi_i^*}{\partial q_i}(q_i^*, q_j^*, H_i^*) = 0$  by definition of  $q_i^*$ , this condition becomes

$$\left( \mathbb{E}[U'(\pi_i^*) \tilde{c}] + \mathbb{E}[U'(\pi_i^*)] \left( -F + P'(Q^*) q_i^* \frac{\partial q_j^*}{\partial H_i} \right) \right) = 0.$$

Dividing by  $\mathbb{E}[U'(\pi_i^*)] > 0$  and selecting  $H_i^* = H_j^* = H^*$  yields equation (12).

2. Equations (12), and (8) yield:

$$\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0 \Leftrightarrow \widehat{c}(q^*, q^*, H) < F \Leftrightarrow H^* > q^*$$

Thus, at a symmetric equilibrium, equation (7) yields

$$P(2q^*) + q^* P'(2q^*) = \widehat{c}(q^*, q^*, H) < F = P(2q^E(F, F)) + q^E(F, F) P'(2q^E(F, F)).$$

Since  $\left(P(2q) + qP'(2q)\right)$  is decreasing by Assumption 1, this condition is equivalent to

$$q^* > q^E(F, F).$$

■

A marginal increase in  $H_i$  has two effects on firm's  $i$  expected utility. First, a direct expected cost effect: the firm substitutes input at known cost  $F$  for input at uncertain cost  $\tilde{c}$ . When taking the risk-adjusted expectation, this substitution is worth  $(\hat{c}(q_i, q_j, H_i) - F)$ . Second, an indirect effect, through the change in the other firm's production:  $\left(P'(Q^*) q_i^* \frac{\partial q_j^*}{\partial H_i}\right)$ . At the equilibrium, both effects exactly cancel out for both firms, which produces equilibrium conditions (12).

Thus, since  $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$  and  $P'(Q^*) q_i^* < 0$ , firms set  $\hat{c}(q_i^*, q_j^*, H_i^*) < F$ . They over hedge, i.e., hedge more than their (anticipated) production, so that their risk-adjusted expected marginal cost is lower than their "true" expected marginal cost  $\mathbb{E}[\tilde{c}] = F$ . This then leads them to become more aggressive, and produce more than if they were perfectly hedged.

Finally, combining first-order conditions (7) and (12) yields:

$$\left(P(Q^*) - F + q^* P'(Q^*)\right)(H^*, H^*) = \left(-P'(Q^*) q^* \frac{\partial q_j^*}{\partial H_i}\right)(H^*, H^*).$$

for  $i = 1, 2$ . Comparing with first-order condition (2) for  $c_i = F$ , an additional term  $\left(-P'(Q^*) q^* \frac{\partial q_j^*}{\partial H_i}\right)$  is added, that captures the strategic impact of firm  $i$ 's hedging on firm  $j$ 's production decision, as in Fudenberg and Tirole (1984).

#### 4 The case of price competition

We now study the case of price competition. An argument similar to the one used for quantity competition shows that risk averse firms committing to their hedging position before making their pricing decision constitutes the only timing where strategic interactions arise. Since the analysis is technically similar to quantity competition, results are stated briefly, and differences with quantity competition are emphasized. In particular, we will see that the strategic impact of hedging on price competition is exactly opposite from the one on quantity competition.

#### 4.1 Demand and constant input costs

Consider two symmetric firms that compete in price. Firm  $i$  faces demand  $D_i = D(p_i, p_j)$ , decreasing in own price and increasing in the other firm's price, and has constant input cost  $c_i$ . With a slight abuse, we use the same notation as for quantity competition. Firm's  $i$  commercial profit is

$$\pi_i^C = \pi^C(p_i, p_j, c_i) = D(p_i, p_j)(p_i - c_i).$$

**Assumption 2**  $D(p_i, p_j)$  is such that, for  $i = 1, 2$ : (i)  $\pi^C$  is concave in its first argument:

$$\frac{\partial^2 \pi^C}{(\partial p_i)^2}(p_i, p_j, c_i) < 0 \text{ for all } (p_i, p_j, c_i),$$

(ii) for all  $(c_i, c_j)$  close enough to the diagonal<sup>3</sup>, the pricing game with constant costs  $c_i$  and  $c_j$  has a unique interior equilibrium  $(p^E(c_i, c_j), p^E(c_j, c_i))$  solution of

$$\left( (p_i - c_i) \frac{\partial D}{\partial p_i} + D \right) (p^E(c_i, c_j), p^E(c_j, c_i)) = 0,$$

(iii) prices are strategic complements:

$$\frac{\partial^2 \pi^C}{\partial p_i \partial p_j}(p^E(c_i, c_j), p^E(c_j, c_i), c_i) > 0 \text{ for all } (c_i, c_j),$$

and (iv) the own price effect on demand is stronger than the other firm's price effect:

$$\frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial p_j} \leq 0 \text{ and } \frac{\partial^2 D_i}{(\partial p_i)^2} + \frac{\partial^2 D_i}{\partial p_i \partial p_j} \leq 0 \text{ for all } (p_i, p_j).$$

Assumption 2 is met for example for a linear Hotelling demand:

$$D_i = D(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2t},$$

in which case equilibrium prices are:

$$p^E(c_i, c_j) = t + \frac{2c_i + c_j}{2}.$$

---

<sup>3</sup> As in the Cournot case,  $|c_1 - c_2|$  must be small enough to avoid a corner equilibrium.

Concavity of the objective function and strategic complementarity of prices are met by many demand functions. Unicity of equilibrium with constant input costs is required to establish unicity with stochastic input costs. We prove in Appendix B.1 that when the own price effect is stronger than the other's price effect, an increase in one firm's cost increases both prices:

$$\frac{\partial p_i^E}{\partial c_i} = \frac{\partial p^E}{\partial c_i}(c_i, c_j) > 0 \text{ and } \frac{\partial p_j^E}{\partial c_i} = \frac{\partial p^E}{\partial c_i}(c_j, c_i) > 0.$$

## 4.2 Random input costs

Firm  $i$ 's profit for input price  $\tilde{c}$  is:

$$\begin{aligned} \pi_i &= \pi(p_i, p_j, H_i; \tilde{c}) = D(p_i, p_j)(p_i - \tilde{c}) + H_i(\tilde{c} - F) \\ &= D(p_i, p_j)(p_i - F) + (H_i - D(p_i, p_j))(\tilde{c} - F). \end{aligned}$$

The expected value for shareholders is

$$v_i = v(p_i, p_j, H_i) = \mathbb{E}[U(\pi(p_i, p_j, H_i; \tilde{c}))].$$

As will be proven below, an equilibrium of the pricing game exists, and, under certain conditions, is unique, hence of the form  $(p^*(H_i, H_j), p^*(H_j, H_i))$ . The expected value of firm  $i$  for its shareholders is

$$V_i = V(H_i, H_j) = v(p^*(H_i, H_j), p^*(H_j, H_i), H_i).$$

Now,

$$\begin{aligned} \frac{\partial v_i}{\partial p_i} &= \mathbb{E}\left[U'(\pi_i) \frac{\partial \pi_i}{\partial p_i}\right] = \mathbb{E}\left[U'(\pi_i) \cdot \left(D_i + (p_i - \tilde{c}) \frac{\partial D_i}{\partial p_i}\right)\right] \\ &= \mathbb{E}[U'(\pi_i)] \left(D_i + (p_i - \hat{c}_i) \frac{\partial D_i}{\partial p_i}\right) \end{aligned}$$

where

$$\hat{c}_i = \hat{c}(p_i, p_j, H_i) \equiv \frac{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c})) \tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]} = F + \frac{\text{cov}[U'(\pi(p_i, p_j, H_i; \tilde{c})), \tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]} \quad (13)$$

$$\frac{\partial^2 v_i}{\partial p_i^2} = \mathbb{E}\left[U''(\pi_i) \left(\frac{\partial \pi_i}{\partial p_i}\right)^2 + U'(\pi_i) \frac{\partial^2 \pi_i}{\partial p_i^2}\right] < 0.$$

Since  $v(p_i, p_j, H_i)$  is concave in  $p_i$ , if an interior Nash equilibrium of the pricing game  $(p_1^*, p_2^*)(H_1, H_2)$  exists, it solves the system of the necessary first-order conditions:

$$(p_i^* - \hat{c}(p_i^*, p_j^*, H_i)) \frac{\partial D}{\partial p_i}(p_i^*, p_j^*) + D(p_i^*, p_j^*) = 0 \quad (14)$$

for  $i = 1, 2$ .

As proven in Appendix B.2, the risk-adjusted expected cost presents similar properties to the Cournot case:

**Lemma 3** *For any  $(p_i, p_j, H_i)$ :*

$$\frac{\partial \hat{c}_i}{\partial H_i} < 0 \text{ and } \hat{c}_i \leq F \Leftrightarrow H_i \geq D(p_i, p_j)$$

and

$$\frac{\partial \hat{c}_i}{\partial p_j} = \frac{\partial D_i}{\partial p_j} \left\{ (p_i - \hat{c}_i) \widehat{\mathbb{E}}[\rho(\pi_i)(\hat{c}_i - \tilde{c})] + \widehat{\mathbb{E}}[\rho(\pi_i)(\hat{c}_i - \tilde{c})^2] \right\}.$$

Finally, for any  $(H_i, H_j)$ :

$$\frac{\partial \hat{c}}{\partial p_i}(p_i^*, p_j^*, H_i) < 0.$$

The equilibrium of the two-stage game is then characterized as follows:

**Proposition 3** *1. For any  $(H_i, H_j)$  close enough to the diagonal, there exists an interior equilibrium of the pricing game. It is characterized by equations (14) for  $i = 1, 2$ .*

*2. If absolute risk aversion is constant, this equilibrium is unique, and a marginal hedging increase by firm  $i$  reduces firm  $j$ 's equilibrium price:*

$$\frac{\partial p_j^*}{\partial H_i} < 0.$$

*3. If an interior equilibrium  $(H_i^*, H_j^*)$  of the hedging game exists, it is characterized by*

$$\left( \hat{c}(p_i^*, p_j^*, H_i) - F - \left( \frac{\frac{\partial D}{\partial p_j}(p_i^*, p_j^*)}{\frac{\partial D}{\partial p_i}(p_i^*, p_j^*)} D(p_i^*, p_j^*) \frac{\partial p_j^*}{\partial H_i} \right) \right) (H_i^*, H_j^*) = 0 \quad (15)$$

for  $i = 1, 2$ .

4. If a symmetric interior equilibrium exists, and absolute risk aversion is constant, hedging softens price competition: firms under-hedge to commit to higher prices than if marginal costs were constant and equal to  $F$ :

$$\widehat{c}(p^*, p^*, H^*) > F \Leftrightarrow H^* < D(p^*, p^*) \Leftrightarrow p^* > p^E(F, F).$$

**Proof.** The proof follows the steps of Propositions 1 and 2. Details are presented in Appendix B.3. The risk-adjusted costs are bounded, thus the set in which we look for a fixed point is compact and convex in  $\mathbb{R}^2$ . Since all functions are continuous, Brouwer's fixed point theorem guarantees the existence of an equilibrium. If absolute risk aversion is constant, then Assumption 2 guarantees unicity of the equilibrium and the direction of the strategic effect. Equation (15) is derived similarly to equation (12). Comparison of equations (15), (13), and (14) shows that hedging softens price competition. ■

Combining the first-order conditions yields:

$$\left( (p^* - F) \frac{\partial D_i}{\partial p_i}(p^*, p^*) + D(p^*, p^*) - \frac{\partial D_i}{\partial p_j}(p^*, p^*) D(p^*, p^*) \frac{\partial p_j^*}{\partial H_i} \right) (H^*, H^*) = 0.$$

Hedging has indeed a strategic effect, captured by the term  $\frac{\partial p_j^*}{\partial H_i}$ . Publicly keeping a portion of their input price exposure uncovered commits firms to raise prices to reduce demand, hence exposure. This commitment then yields a higher equilibrium price :  $p^* > p^E(F, F)$ .

The direction of the strategic effect is reversed compared to Cournot competition: here, firms under-hedge, hence increase the equilibrium price. This stark difference is best understood by comparing the first-order conditions:

$$\widehat{c}_i - F + \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j^*}{\partial H_i} = 0$$

in the Cournot case, and

$$\widehat{c}_i - F + \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j^*}{\partial H_i} = 0$$

in the case of differentiated Bertrand. In both cases, when firm  $i$  increases hedging, firm  $j$  reduces her strategic variable (quantity or price). If firms compete in quantity, when firm  $j$  increases output, he reduces firm's  $i$  profit



$\left(\frac{\partial \pi_i}{\partial q_j} < 0\right)$ , therefore, at the equilibrium, firm  $i$  hedges to set her risk-adjusted expected cost *below*  $F$ , i.e., becomes more aggressive. Conversely, if firms compete in price, when firm  $j$  raises his price, he increases firm  $i$  profit  $\left(\frac{\partial \pi_i}{\partial p_j} > 0\right)$ , hence firm  $i$  hedges to set her risk-adjusted expected cost *above*  $F$ , i.e., becomes less aggressive.

## 5 Robustness of the results

Constant absolute risk aversion implies both that the second-stage equilibrium is unique, and that hedging toughens quantity competition. These results also hold (but are more difficult to prove) if firms' absolute risk aversion indices do not vary too much. We derive in Appendix C some necessary and sufficient conditions for these properties. All can be cast as an upper bound on

$$\left|\frac{\partial \hat{c}_i}{\partial q_j}\right| = q_i \left|P'(Q)\right| \left|\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})]\right|.$$

We observe that  $\left|\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})]\right| = |\text{cov}[\rho(\pi_i), \tilde{c}]|$  is bounded above:

$$\left|\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})]\right| \leq \rho_{\max} \hat{\mathbb{E}}_i[\max(\hat{c}_i - \tilde{c}, 0)] + \rho_{\min} \hat{\mathbb{E}}_i[\min(\hat{c}_i - \tilde{c}, 0)].$$

Since  $\hat{c}_i = \hat{\mathbb{E}}_i[\tilde{c}]$  and  $\hat{c}_i - \tilde{c} = \max(\hat{c}_i - \tilde{c}, 0) + \min(\hat{c}_i - \tilde{c}, 0)$ ,

$$\hat{\mathbb{E}}_i[\hat{c}_i - \tilde{c}] = \hat{\mathbb{E}}_i[\max(\hat{c}_i - \tilde{c}, 0)] + \hat{\mathbb{E}}_i[\min(\hat{c}_i - \tilde{c}, 0)] = 0,$$

thus

$$\left|\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})]\right| \leq (\rho_{\max} - \rho_{\min}) \hat{\mathbb{E}}_i[\max(\hat{c}_i - \tilde{c}, 0)] \leq (\rho_{\max} - \rho_{\min})(\bar{c} - \underline{c}).$$

Let  $R \leq (\rho_{\max} - \rho_{\min})(\bar{c} - \underline{c})$  be the maximum of  $\left|\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})]\right|$ . We have:

$$\left|\frac{\partial \hat{c}_i}{\partial q_j}\right| \leq \bar{q}^E \left|P'(Q)\right| R \leq \bar{q}^E \left|P'(Q)\right| (\rho_{\max} - \rho_{\min})(\bar{c} - \underline{c}).$$

**Proposition 4** *If the variation of firms' absolute risk aversion  $(\rho_{\max} - \rho_{\min})$  is lower than a threshold, that depends on the convexity of inverse demand  $P(\cdot)$ , the equilibrium of the production game is unique and hedging toughens quantity competition. Specifically, a sufficient condition for unicity of the*

equilibrium and  $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$  is

$$(\rho_{\max} - \rho_{\min}) \leq \begin{cases} \frac{\sqrt{3}-1}{\bar{q}^E(\bar{c}-\underline{c})} \text{ if } P''(Q) > 0 \\ \frac{2\sqrt{3}-3}{\bar{q}^E(\bar{c}-\underline{c})} \text{ if } P''(Q) < 0 \\ \frac{1}{\bar{q}^E(\bar{c}-\underline{c})} \text{ if } P''(Q) = 0 \end{cases}.$$

**Proof.** Details of the proof are presented in Appendix C. We derive sufficient conditions for each property, then we determine the highest upper bound such that all three sufficient conditions are simultaneously satisfied. ■

If absolute risk aversion is constant,  $\rho_{\max} = \rho_{\min}$ , these sufficient conditions are met for any inverse demand function and any bounded distribution of input costs. If absolute risk aversion is not constant, these conditions provide an upper bound on  $(\rho_{\max} - \rho_{\min})$  as a function of  $\bar{q}^E$  and  $(\bar{c} - \underline{c})$ .

A similar analysis can be conducted for price competition, although results are more complex in the general case.

## 6 Strategic incentive to commit to a hedging position

So far, we have argued that firms commit to their hedging strategy because their Boards of Directors do not want them to speculate: risk managers are not allowed to significantly deviate from their pre-announced derivatives position. This restriction has clear advantages in terms of monitoring the activity of traders. What is the strategic impact of this commitment?

First, if firms hedge after setting prices (or quantity), they completely eliminate their exposure to input cost. Conversely, when they hedge before they set prices (or quantities), they keep a portion of their exposure open. Their profits' volatility is increased, hence, *ceteris paribus*, their expected utility is decreased.

Second, if firms compete in quantity, commitment yields higher output, hence lower price. Thus we expect commitment yields a Pareto inferior outcome. On the other hand, if firms compete in price, commitment yields higher price, and thus, if total demand is relatively inelastic (e.g., Hotelling competition), higher profits, which may compensate for the reduction in volatility.

In this Section, we refine this intuition. Consider a new game. The timing is now as follows: at  $t = 0$ , firms either Commit ( $C$ ) or Not Commit ( $NC$ ) to their initial hedging strategy. At  $t = 1$ , firms determine and publicly announce

their initial hedging strategy. At  $t = 2$ , they determine and publicly announce their output (or price). Finally, at  $t = 3$ , if they have not committed at  $t = 0$ , they can modify their initial hedging strategy. Then, input cost is realized, and profits are determined.

The expected utility of firm  $i$  that plays strategy  $X_i \in \{C, NC\}$  while firm  $j$  plays strategy  $X_j \in \{C, NC\}$  is  $\Omega(X_i, X_j)$

To focus on the strategic impact of hedging, we continue to assume that (i) there are no transaction costs associated with hedging, and (ii) the forward price  $F$  as well as the expected spot price  $\mathbb{E}[\tilde{c}]$  are equal and constant. Hedging before or after playing the product market game does not modify the (expected) gains.

We assume that the equilibrium of the second stage is unique (this is the case for example if firms have constant risk aversion), and that a unique symmetric equilibrium of the hedging game exists.

**Proposition 5** *1. Not Committing cannot be sustained in equilibrium.*

*Whether firms compete in quantity or in price,*

$$\Omega(C, NC) > \Omega(NC, NC).$$

*2. If firms compete in quantity, Not Committing is Pareto superior*

$$\Omega(NC, NC) > \Omega(C, C).$$

*3. If firms compete à la Hotelling, have a constant absolute risk aversion, and input costs are normally distributed, Committing is Pareto superior to Non Committing*

$$\Omega(C, C) > \Omega(NC, NC)$$

*and is a dominant strategy*

$$\Omega(C, C) > \Omega(NC, C).$$

**Proof.** We first prove point 1 if firms compete in quantity. Suppose firm 2 plays  $NC$ . If firm 1 plays  $NC$ , its expected utility is  $\Omega(NC, NC)$ . Suppose now firm 1 plays  $C$ . At  $t = 3$ , firm 2, which did not commit, optimally selects complete hedging, while firm 1 has committed to  $H_1$ . At  $t = 2$ , both firms select their output. Assuming the equilibrium  $(\bar{q}_1(H_1), \bar{q}_2(H_1))$  is interior, it

is characterized by the first-order conditions:

$$\begin{cases} \bar{q}_2 P'(\bar{q}_1 + \bar{q}_2) + P(\bar{q}_1 + \bar{q}_2) - F = 0 \\ \bar{q}_1 P'(\bar{q}_1 + \bar{q}_2) + P(\bar{q}_1 + \bar{q}_2) - \hat{c}(\bar{q}_1, \bar{q}_2, H_1) = 0 \end{cases}$$

At  $t = 1$ , firm 1 selects  $\bar{H}_1$  to maximize  $Z(H_1) = V(\bar{q}_1(H_1), \bar{q}_2(H_1), H_1)$ . If firm 1 selects  $H_1 = q^E(F, F)$ ,  $q_1 = q_2 = q^E(F, F)$  is a solution of the system, hence is the unique equilibrium for  $H_1 = q^E(F, F)$ . It yields the expected payoff  $\Omega(NC, NC)$ . When firm 2 does not commit, firm 1 can guarantee itself at least  $\Omega(NC, NC)$ , thus  $\Omega(C, NC) \geq \Omega(NC, NC)$ . Then,

$$\begin{aligned} \frac{dZ}{dH_1}(q^E(F, F)) &= \mathbb{E} \left[ U'(\pi_1) \left( \bar{q}_1 P'(\bar{Q}) \frac{d\bar{q}_2}{dH_1} + (\tilde{c} - F) \right) \right] \\ &= \bar{q}_1 P'(\bar{Q}) \frac{d\bar{q}_2}{dH_1} \mathbb{E} [U'(\pi_1)] \end{aligned}$$

since  $\hat{c}(q^E(F, F), q^E(F, F), q^E(F, F)) = F$ . Then, since  $H_1 = \bar{q}_1$ ,  $\frac{\partial \hat{c}_1}{\partial q_2} = 0$ , hence  $\frac{d\bar{q}_2}{dH_1} < 0$ . Thus,  $\frac{dZ}{dH_1}(q^E(F, F)) > 0$ , hence for  $\varepsilon > 0$  arbitrarily small,  $Z(q^E(F, F) + \varepsilon) > Z(q^E(F, F)) \geq \Omega(NC, NC)$ . Thus:

$$\Omega(C, NC) > \Omega(NC, NC).$$

The proof of point 1 proceeds along the same lines if firms compete in price, and is presented in Appendix D, along with the formal proof of the other points. As expected, when firms compete in quantity,  $(C, C)$  yields lower price and higher volatility, hence lower expected utility than  $(NC, NC)$ . On the other hand, if firms compete à la Hotelling, the expected profit increase compensates for the loss coming from increased volatility, hence (i) Committing is Pareto superior<sup>4</sup>:  $\Omega(C, C) > \Omega(NC, NC)$ , and (ii) is the best response to the other firm's Committing:  $\Omega(C, C) > \Omega(NC, C)$ . ■

The first point is striking: even though both firms Non Committing is Pareto superior for firms competing in quantity, an individual firm prefers to Commit when the other one does not, since she can always do strictly better than replicating the Non Committing outcome.

Thus, whether firms compete in quantity or in price, they have a strategic incentive to commit.

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<sup>4</sup>Yields a higher expected utility.

## 7 Concluding remarks

This article examines how firms facing volatile input prices and holding some degree of market power in their product market link their risk management with their production or pricing strategies. This issue is relevant in many industries ranging from manufacturing to energy retailing, where risk averse firms decide on their hedging strategies *before* their product market strategies. We find that hedging modifies the pricing and production strategies of firms. This strategic effect is channelled through the risk-adjusted expected cost, i.e., the expected marginal cost under the measure induced by shareholders' risk aversion. It has diametrically opposed impacts depending on the nature of product market competition: hedging toughens quantity competition while it softens price competition. Finally, committing to a hedging strategy is always a best response to non committing, and is a dominant strategy if firms compete à la Hotelling.

This work can be expanded in many directions. First, it would be interesting to endogenize pricing flexibility. We have assumed that industry practices dictate whether prices are flexible or not. This is true in practice. However, it would be interesting to know under what conditions price flexibility is indeed an equilibrium outcome.

Second, it would be interesting to examine asymmetric situations, such as when one firm is market leader and announces its hedging strategy before the other, or when different firms have different costs.

Finally, it would be interesting to test empirically these models' predictions, in particular whether firms incorporate their and their competitors' hedging in their pricing strategies. The airlines industry appears to offer fertile ground for such an analysis: airlines face volatile fuel cost, and appear to have retained some pricing power, at least on some routes. Furthermore, as evidenced by the rich academic literature on that industry (e.g., Carter, Rogers, and Simkins (2006) and (Gerardi and Shapiro (2009)) data on prices and hedging strategies are publicly available.

## A Quantity competition

### A.1 Properties of the risk-adjusted expected cost (Lemma 2)

For any  $(q_i, q_j, H_i)$ ,

$$\begin{aligned}
\frac{\partial \hat{c}_i}{\partial H_i} &= \frac{\mathbb{E}[U'(\pi_i)] \mathbb{E}\left[U''(\pi_i) \frac{\partial \pi_i}{\partial H_i} \tilde{c}\right] - \mathbb{E}[U'(\pi_i) \tilde{c}] \mathbb{E}\left[U''(\pi_i) \frac{\partial \pi_i}{\partial H_i}\right]}{(\mathbb{E}[U'(\pi_i)])^2} \\
&= -\frac{\mathbb{E}\left[U'(\pi_i) \left(-\frac{U''(\pi_i)}{U'(\pi_i)} \frac{\partial \pi_i}{\partial H_i} \tilde{c}\right)\right]}{\mathbb{E}[U'(\pi_i)]} + \frac{\mathbb{E}[U'(\pi_i) \tilde{c}]}{\mathbb{E}[U'(\pi_i)]} \frac{\mathbb{E}\left[U'(\pi_i) \left(-\frac{U''(\pi_i)}{U'(\pi_i)} \frac{\partial \pi_i}{\partial H_i}\right)\right]}{\mathbb{E}[U'(\pi_i)]} \\
&= -\widehat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial H_i} \tilde{c} \right] + \hat{c}_i \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial H_i} \right] \\
&= \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial H_i} (\hat{c}_i - \tilde{c}) \right]
\end{aligned}$$

Then, since  $\frac{\partial \pi_i}{\partial H_i} = (\tilde{c} - F)$ ,

$$\begin{aligned}
\frac{\partial \hat{c}_i}{\partial H_i} &= \widehat{\mathbb{E}}_i [\rho(\pi_i) (\tilde{c} - F) (\hat{c}_i - \tilde{c})] = -\widehat{\mathbb{E}}_i [\rho(\pi_i) (\tilde{c} - \hat{c}_i)^2] + (\hat{c}_i - F) \widehat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})] \\
&= -\left( \widehat{\mathbb{E}}_i [\rho(\pi_i) (\tilde{c} - \hat{c}_i)^2] + \frac{\text{cov}[U'(\pi_i), \tilde{c}] \cdot \widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}]}{\mathbb{E}[U'(\pi_i)]} \right)
\end{aligned}$$

since

$$\hat{c}_i - F = \frac{\mathbb{E}[U'(\pi_i) \tilde{c}]}{\mathbb{E}[U'(\pi_i)]} - \mathbb{E}[\tilde{c}] = \frac{\text{cov}[U'(\pi_i), \tilde{c}]}{\mathbb{E}[U'(\pi_i)]}$$

and

$$\widehat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})] = \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) (\widehat{\mathbb{E}}_i[\tilde{c}] - \tilde{c}) \right] = -\widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}].$$

Since (i)  $\rho(\cdot)$  and  $U'(\cdot)$  are both non-increasing in  $\pi_i$ , and (ii)  $\pi_i$  increases in  $\tilde{c}$  if and only if  $H_i > q_i$ , we have (i)  $\text{cov}[U'(\pi_i), \tilde{c}] \cdot \widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}] \geq 0$ , hence  $\frac{\partial \hat{c}_i}{\partial H_i} < 0$ , and  $\hat{c}_i \leq F \Leftrightarrow H_i \geq q_i$ . This establishes (8).

Similar algebra yields

$$\begin{aligned}
\frac{\partial \hat{c}_i}{\partial q_j} &= \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial q_j} (\hat{c}_i - \tilde{c}) \right] \\
&= q_i P'(Q) \widehat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})] \\
&= -q_i P'(Q) \widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}]
\end{aligned}$$

which establishes (9). (10) follows from (9).

Finally,

$$\frac{\partial \hat{c}_i}{\partial q_i} = \hat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial q_i} (\hat{c}_i - \tilde{c}) \right].$$

For any  $(H_i, H_j)$ , the equilibrium  $(q_i^*, q_j^*) (H_i, H_j)$  of the Cournot game satisfies

$$P(q_i^* + q_j^*) + q_i^* P'(q_i^* + q_j^*) = \hat{c}(q_i^*, q_j^*, H_i).$$

Thus,

$$\frac{\partial \pi}{\partial q_i}(q_i^*, q_j^*, H_i; \tilde{c}) = P(q_i^* + q_j^*) + q_i^* P'(q_i^* + q_j^*) - \tilde{c} = \hat{c}(q_i^*, q_j^*, H_i) - \tilde{c},$$

and

$$\frac{\partial \hat{c}}{\partial q_i}(q_i^*, q_j^*, H_i) = \hat{\mathbb{E}}_i \left[ \rho(\pi(q_i^*, q_j^*, H_i)) (\hat{c}(q_i^*, q_j^*, H_i) - \tilde{c})^2 \right] > 0$$

which establishes (11).

## A.2 Unicity of equilibrium (Proposition 1)

As mentioned in the main text, the equilibrium of the production game  $(q_1^*, q_2^*) (H_1, H_2)$  is unique if the real parts of the eigenvalues of the Jacobian  $J(q_1^*, q_2^*, H_1, H_2)$  are negative, where

$$J(q_1^*, q_2^*, H_1, H_2) = \begin{bmatrix} \frac{\partial q_1^E}{\partial q_1} - 1 & \frac{\partial q_1^E}{\partial q_2} \\ \frac{\partial q_2^E}{\partial q_1} & \frac{\partial q_2^E}{\partial q_2} - 1 \end{bmatrix}.$$

This is an application of Lyapunov's stability theorem (see for example Khalil (2002)). The eigenvalues are the roots of:

$$\lambda^2 - \lambda Tr + Det = 0$$

where  $Tr$  is the trace of  $J$  and  $Det$  its determinant. The roots are:

$$\lambda_{\pm} = \frac{Tr \pm \sqrt{Tr^2 - 4Det}}{2}.$$

If  $Tr^2 - 4Det < 0$ , the two roots are complex and conjugate. Their real part is negative if and only if  $Tr < 0$ . If  $Tr^2 - 4Det \geq 0$ , the two roots are real.  $Tr + \sqrt{Tr^2 - 4Det} < 0$  requires  $Tr < 0$ . Then, it also requires  $Det > 0$ .

Thus, we require  $Tr < 0$  and  $Det > 0$ .

$$Tr = \left( \frac{\partial q_1^E}{\partial q_1} + \frac{\partial q_2^E}{\partial q_2} - 2 \right).$$

By definition,

$$q_i^E = q^E(\hat{c}(q_i, q_j, H_i), \hat{c}(q_j, q_i, H_j), H_i),$$

for  $i = 1, 2$ , thus

$$\frac{\partial q_i^E}{\partial q_i} = \frac{\partial q_i^E}{\partial c_i} \frac{\partial \hat{c}_i}{\partial q_i} + \frac{\partial q_i^E}{\partial c_j} \frac{\partial \hat{c}_j}{\partial q_i}, \quad (16)$$

and

$$\frac{\partial q_i^E}{\partial q_j} = \frac{\partial q_i^E}{\partial c_i} \frac{\partial \hat{c}_i}{\partial q_j} + \frac{\partial q_i^E}{\partial c_j} \frac{\partial \hat{c}_j}{\partial q_j}. \quad (17)$$

Thus,

$$Tr = \frac{\partial q_1^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_1} + \frac{\partial q_1^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_1} + \frac{\partial q_2^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_2} + \frac{\partial q_2^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_2} - 2.$$

From Lemma 2,

$$\rho \text{ constant} \Rightarrow \frac{\partial \hat{c}_1}{\partial q_2} = \frac{\partial \hat{c}_2}{\partial q_1} = 0$$

$\Rightarrow$

$$Tr(q_1^*, q_2^*, H_1, H_2) = \left( \frac{\partial q_1^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_1} + \frac{\partial q_2^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_2} \right) (q_1^*, q_2^*, H_1, H_2) - 2 < -2 < 0.$$

We now examine  $Det(q_1^*, q_2^*, H_1, H_2)$ .

$$\begin{aligned} Det &= \left( \frac{\partial q_1^E}{\partial q_1} - 1 \right) \left( \frac{\partial q_2^E}{\partial q_2} - 1 \right) - \frac{\partial q_1^E}{\partial q_2} \frac{\partial q_2^E}{\partial q_1} \\ &= \frac{\partial q_1^E}{\partial q_1} \frac{\partial q_2^E}{\partial q_2} - \frac{\partial q_1^E}{\partial q_2} \frac{\partial q_2^E}{\partial q_1} - \frac{\partial q_1^E}{\partial q_1} - \frac{\partial q_2^E}{\partial q_2} + 1 \end{aligned}$$

Substituting in  $\frac{\partial q_i^E}{\partial q_i}$  and  $\frac{\partial q_i^E}{\partial q_j}$  from equations (16) and (17), then simplifying yields

$$\frac{\partial q_1^E}{\partial q_1} \frac{\partial q_2^E}{\partial q_2} - \frac{\partial q_1^E}{\partial q_2} \frac{\partial q_2^E}{\partial q_1} = \left( \frac{\partial q_1^E}{\partial c_1} \frac{\partial q_2^E}{\partial c_2} - \frac{\partial q_1^E}{\partial c_2} \frac{\partial q_2^E}{\partial c_1} \right) \left( \frac{\partial \hat{c}_1}{\partial q_1} \frac{\partial \hat{c}_2}{\partial q_2} - \frac{\partial \hat{c}_1}{\partial q_2} \frac{\partial \hat{c}_2}{\partial q_1} \right).$$

Substituting in  $\frac{\partial q_i^E}{\partial c_i}$  and  $\frac{\partial q_j^E}{\partial c_i}$  from equations (3) and (4), then simplifying



yields

$$\frac{\partial q_1^E}{\partial c_1} \frac{\partial q_2^E}{\partial c_2} - \frac{\partial q_1^E}{\partial c_2} \frac{\partial q_2^E}{\partial c_1} = \frac{1}{P'(Q^E) \left( 3P'(Q^E) + QP''(Q^E) \right)}.$$

Thus,

$$Det = \frac{\frac{\partial \hat{c}_1}{\partial q_1} \frac{\partial \hat{c}_2}{\partial q_2} - \frac{\partial \hat{c}_1}{\partial q_2} \frac{\partial \hat{c}_2}{\partial q_1}}{P'(Q^E) \left( 3P'(Q^E) + Q^E P''(Q^E) \right)} - Tr - 1.$$

Then,

$$\rho \text{ constant} \Rightarrow \frac{\partial \hat{c}_2}{\partial q_1} = \frac{\partial \hat{c}_1}{\partial q_2} = 0$$

$\Rightarrow$

$$Det(q_1^*, q_2^*, H_1, H_2) = \left( \frac{\frac{\partial \hat{c}_1}{\partial q_1} \frac{\partial \hat{c}_2}{\partial q_2}}{P'(Q^E) \left( 3P'(Q^E) + Q^* P''(Q^E) \right)} - Tr \right) (q_1^*, q_2^*, H_1, H_2) - 1 \\ > -Tr(q_1^*, q_2^*, H_1, H_2) - 1 > 1.$$

Finally, since the equilibrium is unique, it takes the form  $q_1^* = q^*(H_1, H_2)$  and  $q_2^* = q^*(H_2, H_1)$ . The proof proceeds by contradiction. Suppose  $q_1 = \lambda(H_1, H_2)$  and  $q_2 = \mu(H_2, H_1)$  constitute an equilibrium, with  $\lambda(.,.) \neq \mu(.,.)$ . Then,

$$\begin{cases} q_1 = \lambda(H_1, H_2) = q^E(\hat{c}(\lambda(H_1, H_2), \mu(H_2, H_1), H_1), \hat{c}(\mu(H_2, H_1), \lambda(H_1, H_2), H_2))) \\ q_2 = \mu(H_2, H_1) = q^E(\hat{c}(\mu(H_2, H_1), \lambda(H_1, H_2), H_2), \hat{c}(\lambda(H_1, H_2), \mu(H_2, H_1), H_1))) \end{cases}.$$

If we switch  $H_1$  and  $H_2$ ,  $q_1 = \lambda(H_2, H_1)$  and  $q_2 = \mu(H_1, H_2)$  is also an equilibrium. Thus,

$$\begin{cases} q_1 = \lambda(H_2, H_1) = q^E(\hat{c}(\lambda(H_2, H_1), \mu(H_1, H_2), H_2), \hat{c}(\mu(H_1, H_2), \lambda(H_2, H_1), H_1))) \\ q_2 = \mu(H_1, H_2) = q^E(\hat{c}(\mu(H_1, H_2), \lambda(H_2, H_1), H_1), \hat{c}(\lambda(H_2, H_1), \mu(H_1, H_2), H_2))) \end{cases}.$$

Thus,  $q_1 = \mu(H_1, H_2)$  and  $q_2 = \lambda(H_2, H_1)$  also constitutes an equilibrium, which contradicts the unicity of the equilibrium. Thus,  $\lambda(.,.) = \mu(.,.) = q^*(.,.)$ .

### A.3 Impact of $H_i$ on $q_j^*$ with constant absolute risk aversion (Proposition 1)

Define

$$\psi(q_i, q_j, H_i) = P(Q) - \hat{c}(q_i, q_j, H_i) + q_i P'(Q)$$

The necessary and sufficient conditions characterizing the unique equilibrium of the production game are

$$\psi(q_i^*, q_j^*, H_i) = \psi(q_j^*, q_i^*, H_j) = 0.$$

Total differentiation of the First Order Conditions with respect to  $H_i$  yields:

$$\begin{cases} \left( \psi_1 \frac{\partial q_i^*}{\partial H_i} + \psi_2 \frac{\partial q_j^*}{\partial H_i} + \psi_3 \right) (q_i^*, q_j^*, H_i) = 0 \\ \left( \psi_1 \frac{\partial q_j^*}{\partial H_i} + \psi_2 \frac{\partial q_i^*}{\partial H_i} \right) (q_j^*, q_i^*, H_j) = 0 \end{cases},$$

where

$$\begin{cases} \psi_1(q_i, q_j, H_i) = 2P'(q_i + q_j) + q_i P''(q_i + q_j) - \frac{\partial \hat{c}_i}{\partial q_i} \\ \psi_2(q_i, q_j, H_i) = P'(q_i + q_j) + q_i P''(q_i + q_j) - \frac{\partial \hat{c}_i}{\partial q_j} \\ \psi_3(q_i, q_j, H_i) = -\frac{\partial \hat{c}_i}{\partial H_i} > 0 \end{cases}.$$

Thus, assuming

$$\Delta = \psi_1(q_i^*, q_j^*, H_i) \psi_1(q_j^*, q_i^*, H_j) - \psi_2(q_i^*, q_j^*, H_i) \psi_2(q_j^*, q_i^*, H_j) \neq 0,$$

$$\begin{cases} \frac{\partial q_i^*}{\partial H_i}(H_i, H_j) = -\frac{\psi_1(q_j^*, q_i^*, H_j)}{\Delta} \psi_3(q_i^*, q_j^*, H_i) \\ \frac{\partial q_j^*}{\partial H_i}(H_j, H_i) = \frac{\psi_2(q_j^*, q_i^*, H_j)}{\Delta} \psi_3(q_i^*, q_j^*, H_i) \end{cases}.$$

If  $\rho$  is constant,  $\frac{\partial \hat{c}_i}{\partial q_j} = 0$ , thus

$$\psi_2(q_i, q_j, H_i) = P'(q_i + q_j) + q_i P''(q_i + q_j) < 0.$$

Then:

$$\begin{aligned} \Delta &= \left( 2P'(Q^*) + q_i^* P''(Q^*) - \frac{\partial \hat{c}_i}{\partial q_i} \right) \left( 2P'(Q^*) + q_j^* P''(Q^*) - \frac{\partial \hat{c}_j}{\partial q_j} \right) \\ &\quad - \left( P'(Q^*) + q_i P''(Q^*) \right) \left( P'(Q^*) + q_j P''(Q^*) \right) \\ &= \left( P'(Q^*) - \frac{\partial \hat{c}_i}{\partial q_i} \right) \left( P'(Q^*) - \frac{\partial \hat{c}_j}{\partial q_j} \right) + \left( P'(Q^*) - \frac{\partial \hat{c}_i}{\partial q_i} \right) \left( P'(Q^*) + q_j^* P''(Q^*) \right) \\ &\quad + \left( P'(Q^*) - \frac{\partial \hat{c}_j}{\partial q_j} \right) \left( P'(Q^*) + q_i^* P''(Q^*) \right). \end{aligned}$$

Since all terms in parentheses are negative,  $\Delta > 0$ , thus

$$\frac{\partial q_j^*}{\partial H_i} = \frac{\partial q^*}{\partial H_i}(H_j, H_i) = \frac{\psi_2(q_j^*, q_i^*, H_i)}{\Delta} \psi_3(q_i^*, q_j^*, H_i) < 0.$$

## B Price competition

Define

$$\psi(p_i, p_j, H_i) = \frac{\partial D}{\partial p_i}(p_i, p_j)(p_i - \hat{c}(p_i, p_j, H_i)) + D(p_i, p_j)$$

where

$$\hat{c}_i \equiv \hat{c}(p_i, p_j, H_i) \equiv \frac{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))\tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]}$$

is the risk-adjusted expected cost of firm  $i$ . Suppose a unique interior equilibrium of the pricing game  $(p_1^*, p_2^*)(H_1, H_2)$  exists. The first-order conditions characterizing the equilibrium are

$$\psi(p_i^*, p_j^*, H_i) = \psi(p_j^*, p_i^*, H_j) = 0.$$

Assuming

$$\Delta = \psi_1(p_i^*, p_j^*, H_i)\psi_1(p_j^*, p_i^*, H_j) - \psi_2(p_i^*, p_j^*, H_i)\psi_2(p_j^*, p_i^*, H_j) \neq 0,$$

$$\begin{cases} \frac{\partial p_i^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_i, H_j) = -\frac{\psi_1(p_j^*, p_i^*, H_j)}{\Delta}\psi_3(p_i^*, p_j^*, H_i) \\ \frac{\partial p_j^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_j, H_i) = \frac{\psi_2(p_j^*, p_i^*, H_j)}{\Delta}\psi_3(p_i^*, p_j^*, H_i) \end{cases}.$$

where

$$\begin{cases} \psi_1(p_i, p_j, H_i) = 2\frac{\partial D_i}{\partial p_i} + (p_i - \hat{c}_i)\frac{\partial^2 D_i}{(\partial p_i)^2} - \frac{\partial D_i}{\partial p_i}\frac{\partial \hat{c}_i}{\partial p_i} \\ \psi_2(p_i, p_j, H_i) = \frac{\partial D_i}{\partial p_j} + (p_i - \hat{c}_i)\frac{\partial^2 D_i}{\partial p_i \partial p_j} - \frac{\partial D_i}{\partial p_i}\frac{\partial \hat{c}_i}{\partial p_j} \\ \psi_3(p_i, p_j, H_i) = -\frac{\partial D_i}{\partial p_i}\frac{\partial \hat{c}_i}{\partial H_i} \end{cases}.$$

### B.1 Impact of $c_i$ on $p_i^E$ and $p_j^E$ (constant input costs)

Suppose first the marginal costs are constant:

$$\psi^E(p_i, p_j, c_i) = \frac{\partial D(p_i, p_j)}{\partial p_i}(p_i - c_i) + D(p_i, p_j)$$

and

$$\begin{cases} \psi_1^E(p_i, p_j, c_i) = 2\frac{\partial D_i}{\partial p_i} + (p_i - c_i)\frac{\partial^2 D_i}{(\partial p_i)^2} \\ \psi_2^E(p_i, p_j, c_i) = \frac{\partial D_i}{\partial p_j} + (p_i - c_i)\frac{\partial^2 D_i}{\partial p_i \partial p_j} \\ \psi_3^E(p_i, p_j, c_i) = -\frac{\partial D_i}{\partial p_i} \end{cases}.$$

Assumption 2 guarantees (i) existence and unicity of an equilibrium  $(p^E(c_i, c_j), p^E(c_j, c_i))$ ,

(ii)  $\psi_1^E(p_i, p_j, c_i) < 0$ , since  $\pi^C(p_i, p_j, c_i)$  is concave in  $p_i$ , (iii)

$$\psi_2^E(p^E(c_i, c_j), p^E(c_j, c_i), c_i) > 0$$

since prices are strategic complements, and (iv)

$$(\psi_1^E + \psi_2^E)(p^E(c_i, c_j), p^E(c_j, c_i), c_i) < 0$$

since the own price effect dominates. Thus:

$$\Delta^E = \psi_1^E(p_i^E, p_j^E, c_i) \psi_1^E(p_j^E, p_i^E, c_j) - \psi_2^E(p_i^E, p_j^E, c_i) \psi_2^E(p_j^E, p_i^E, c_j) > 0$$

and

$$\begin{cases} \frac{\partial p_i^E}{\partial c_i} = \frac{\partial p^E}{\partial c_i}(c_i, c_j) = \frac{\partial D_i}{\partial p_i} \frac{\psi_1^E(p_j^E, p_i^E, c_j)}{\Delta^E} = \frac{\partial D_i}{\partial p_i} \frac{2 \frac{\partial D_j}{\partial p_j} + (p_j - c_j) \frac{\partial^2 D_j}{(\partial p_j)^2}}{\Delta^E} > 0 \\ \frac{\partial p_j^E}{\partial c_i} = \frac{\partial p^E}{\partial c_i}(c_j, c_i) = -\frac{\partial D_i}{\partial p_i} \frac{\psi_2^E(p_j^E, p_i^E, c_j)}{\Delta^E} = -\frac{\partial D_i}{\partial p_i} \frac{\frac{\partial D_j}{\partial p_i} + (p_j - c_j) \frac{\partial^2 D_j}{\partial p_i \partial p_j}}{\Delta^E} > 0 \end{cases}.$$

## B.2 Properties of the risk-adjusted expected cost (Lemma 3)

For any  $(p_i, p_j, H_i)$ , the same derivation as for Cournot competition yields:

$$\begin{aligned} \frac{\partial \hat{c}_i}{\partial H_i} &= \hat{\mathbb{E}}_i[\rho(\pi_i)(\tilde{c} - F)(\hat{c}_i - \tilde{c})] \\ &= -\hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})^2] + (\hat{c}_i - F) \hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})] \\ &= -\left( \hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})^2] + \frac{\text{cov}[U'(\pi_i), \tilde{c}] \cdot \widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}]}{\mathbb{E}[U'(\pi_i)]} \right). \end{aligned}$$

Since (i)  $\rho(\cdot)$  and  $U'(\cdot)$  are both non-increasing, and (ii)  $\pi_i$  increases in  $\tilde{c}$  if and only if  $H_i > D(p_i, p_j)$ , we have (i)  $\text{cov}[U'(\pi_i), \tilde{c}] \cdot \widehat{\text{cov}}_i[\rho(\pi_i), \tilde{c}] \geq 0$ , thus  $\frac{\partial \hat{c}_i}{\partial H_i} < 0$ , and (ii) and  $\hat{c}_i \leq F \Leftrightarrow H_i \geq D(p_i, p_j)$ .

Similarly:

$$\begin{aligned} \frac{\partial \hat{c}_i}{\partial p_j} &= \hat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial p_j} (\hat{c}_i - \tilde{c}) \right] \\ &= \frac{\partial D_i}{\partial p_j} \hat{\mathbb{E}}_i[\rho(\pi_i)(p_i - \tilde{c})(\hat{c}_i - \tilde{c})] \\ &= \frac{\partial D_i}{\partial p_j} \left\{ (p_i - \hat{c}_i) \hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})] + \hat{\mathbb{E}}_i[\rho(\pi_i)(\hat{c}_i - \tilde{c})^2] \right\}. \end{aligned}$$

Thus:

$$\frac{\partial \hat{c}}{\partial p_j}(p_i^*, p_j^*, H_i) = \left( \frac{\partial D_i}{\partial p_j} \left\{ \frac{D_i}{\frac{\partial D_i}{\partial p_i}} \widehat{cov}_i[\rho(\pi_i), \tilde{c}] + \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) (\hat{c}_i - \tilde{c})^2 \right] \right\} \right) (p_i^*, p_j^*, H_i)$$

Then:

$$H_i > D(p_i^*, p_j^*) \Leftrightarrow \widehat{cov}_i[\rho(\pi_i), \tilde{c}] < 0 \Rightarrow \frac{\partial \hat{c}}{\partial p_j}(p_i^*, p_j^*, H_i) > 0.$$

Finally:

$$\frac{\partial \hat{c}_i}{\partial p_i} = \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) \frac{\partial \pi_i}{\partial p_i} (\hat{c}_i - \tilde{c}) \right].$$

Then,

$$\frac{\partial \hat{c}}{\partial p_i}(p_i^*, p_j^*, H_i) = \left( \frac{\partial D}{\partial p_i} \widehat{\mathbb{E}}_i \left[ \rho(\pi_i) (\hat{c}_i - \tilde{c})^2 \right] \right) (p_i^*, p_j^*, H_i) < 0$$

since

$$\frac{\partial \pi}{\partial p_i}(p_i^*, p_j^*, H_i) = \left( (\hat{c}_i - \tilde{c}) \frac{\partial D_i}{\partial p_i} \right) (p_i^*, p_j^*, H_i).$$

### B.3 Characterization of the equilibrium (Proposition 3)

#### B.3.1 Existence and unicity of equilibrium of the pricing game

Since we ultimately focus on symmetric equilibria, we assume that  $|H_i - H_j|$  is small enough that the equilibrium is interior. The equilibrium prices are therefore fixed points of the function  $\Phi$  defined from  $\mathbb{R}^2$  into  $\mathbb{R}^2$  by

$$\begin{cases} \Phi_1(p_1, p_2) = p^E(\hat{c}(p_1, p_2, H_1), \hat{c}(p_2, p_1, H_2)) \\ \Phi_2(p_1, p_2) = p^E(\hat{c}(p_2, p_1, H_2), \hat{c}(p_1, p_2, H_1)) \end{cases}.$$

Since  $\tilde{c} \leq \bar{c}$  by assumption,

$$\hat{c}(p_i, p_j, H_i) = \frac{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c})) \tilde{c}]}{\mathbb{E}[U'(\pi(p_i, p_j, H_i; \tilde{c}))]} \leq \bar{c}$$

for all  $(p_i, p_j, H_i)$ . Thus, since  $\frac{\partial p^E}{\partial c_i} \geq 0$  and  $\frac{\partial p^E}{\partial c_j} \geq 0$ ,

$$p^E(\hat{c}(p_i, p_j, H_i), \hat{c}(p_j, p_i, H_j)) \leq p^E(\bar{c}, \bar{c}) \equiv \overline{p^E}$$

Since  $p_i \geq 0$  by definition, we can limit our search for a fixed point to the compact and convex set  $[0, \overline{p^E}]^2$ . Brouwer's theorem then applies, and there exists a fixed point, i.e., an equilibrium.

This equilibrium  $(p_1^*, p_2^*)$  is unique if  $Tr < 0$  and  $Det > 0$ . Then:

$$Tr(p_1^*, p_2^*, H_1, H_2) = \left( \frac{\partial p_1^E}{\partial p_1} + \frac{\partial p_2^E}{\partial p_2} - 2 \right) (p_1^*, p_2^*, H_1, H_2) = A(p_1^*, p_2^*, H_1, H_2) - 2.$$

where

$$A = \frac{\partial p_1^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial p_1} + \frac{\partial p_1^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial p_1} + \frac{\partial p_2^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial p_2} + \frac{\partial p_2^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial p_2}.$$

$$\frac{\partial p_i^E}{\partial c_i} \frac{\partial \hat{c}_i}{\partial p_i} (p_i^*, p_j^*, H_i, H_j) = \left( \left( \frac{\partial D_i}{\partial p_i} \right)^2 \frac{\psi_1^E(p_j^E, p_i^E, c_j) \hat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})^2]}{\Delta^E} \right) (p_i^*, p_j^*, H_i),$$

if  $\rho$  is constant,

$$\frac{\partial \hat{c}_i}{\partial p_j} = \frac{\partial D_i}{\partial p_j} \hat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})^2],$$

thus

$$\frac{\partial p_j^E}{\partial c_i} \frac{\partial \hat{c}_i}{\partial p_j} = - \frac{\partial D_i}{\partial p_i} \frac{\partial D_i}{\partial p_j} \frac{\psi_2^E(p_j^E, p_i^E, c_j) \hat{\mathbb{E}}_i [\rho(\pi_i) (\hat{c}_i - \tilde{c})^2]}{\Delta^E}.$$

Thus,

$$A(p_1^*, p_2^*, H_1, H_2) = \left( \frac{\frac{\partial D_1}{\partial p_1} \hat{\mathbb{E}}_1 [\rho(\pi_1) (\hat{c}_1 - \tilde{c})^2]}{\Delta^E} \left( \frac{\partial D_1}{\partial p_1} \psi_1^E(p_2, p_1, \hat{c}_2) - \frac{\partial D_1}{\partial p_2} \psi_2^E(p_2, p_1, \hat{c}_2) \right) + \frac{\frac{\partial D_2}{\partial p_2} \hat{\mathbb{E}}_2 [\rho(\pi_2) (\hat{c}_2 - \tilde{c})^2]}{\Delta^E} \left( \frac{\partial D_2}{\partial p_2} \psi_1^E(p_1, p_2, \hat{c}_1) - \frac{\partial D_2}{\partial p_1} \psi_2^E(p_1, p_2, \hat{c}_1) \right) \right) (p_1^*, p_2^*, H_1, H_2)$$

By Assumption 2,  $(\psi_1^E + \psi_2^E)(p_1^*, p_2^*, H_1, H_2) < 0$  and  $\left( \frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial p_j} \right) (p_i, p_j) < 0$ , thus

$$\left( \frac{\partial D_i}{\partial p_i} \psi_1^E - \frac{\partial D_i}{\partial p_j} \psi_2^E \right) (p_1^*, p_2^*, H_1, H_2) > 0,$$

hence  $A(p_1^*, p_2^*, H_1, H_2) < 0$ , hence  $Tr(p_1^*, p_2^*, H_1, H_2) < -2 < 0$ .

We now examine the sign of  $Det(p_1^*, p_2^*, H_1, H_2)$ .

$$Det = \left( \frac{\partial p_1^E}{\partial p_1} - 1 \right) \left( \frac{\partial p_2^E}{\partial p_2} - 1 \right) - \frac{\partial p_2^E}{\partial p_1} \frac{\partial p_1^E}{\partial p_2}$$

The same algebra as in the Cournot case yields

$$Det = \left( \frac{\partial p_1^E}{\partial c_1} \frac{\partial p_2^E}{\partial c_2} - \frac{\partial p_1^E}{\partial c_2} \frac{\partial p_2^E}{\partial c_1} \right) \left( \frac{\partial c_1}{\partial p_1} \frac{\partial \hat{c}_2}{\partial p_2} - \frac{\partial \hat{c}_2}{\partial p_1} \frac{\partial \hat{c}_1}{\partial p_2} \right) - Tr - 1.$$

Define

$$B = \left( \frac{\partial p_i^E}{\partial c_i} \frac{\partial p_j^E}{\partial c_j} - \frac{\partial p_j^E}{\partial c_i} \frac{\partial p_i^E}{\partial c_j} \right).$$

Analysis of the constant cost case yields

$$B = \frac{1}{(\Delta^E)^2} \begin{bmatrix} \psi_1^E(p_j^E, p_i^E, c_j) \psi_3^E(p_i^E, p_j^E, c_i) \psi_1^E(p_i^E, p_j^E, c_i) \psi_3^E(p_j^E, p_i^E, c_j) \\ -\psi_2^E(p_j^E, p_i^E, c_j) \psi_3^E(p_i^E, p_j^E, c_i) \psi_2^E(p_i^E, p_j^E, c_i) \psi_3^E(p_j^E, p_i^E, c_j) \end{bmatrix} \\ = \frac{\psi_3^E(p_i^E, p_j^E, H_i) \psi_3^E(p_j^E, p_i^E, H_j)}{\Delta^E} = \frac{\frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j}}{\Delta^E} > 0.$$

Analysis of the risk-adjusted expect cost shows that, if  $\rho$  is constant,

$$\left( \frac{\partial c_1}{\partial p_1} \frac{\partial \hat{c}_2}{\partial p_2} - \frac{\partial \hat{c}_2}{\partial p_1} \frac{\partial \hat{c}_1}{\partial p_2} \right) (p_1^*, p_2^*, H_1, H_2) = \widehat{\mathbb{E}} \left[ \rho(\pi_1) (\hat{c}_1 - \check{c})^2 \right] \widehat{\mathbb{E}} \left[ \rho(\pi_2) (\hat{c}_2 - \check{c})^2 \right] \times \\ \left( \frac{\partial D_1}{\partial p_1} \frac{\partial D_2}{\partial p_2} - \frac{\partial D_1}{\partial p_2} \frac{\partial D_2}{\partial p_1} \right) > 0.$$

Thus,  $\text{Det}(p_1^*, p_2^*, H_1, H_2) > 0$ .

### B.3.2 Equilibrium of the hedging game

Suppose a unique interior of the hedging game exists. For  $i = 1, 2$ , the first order conditions defining this equilibrium are

$$\mathbb{E} \left[ U'(\pi_i^*) \left( \frac{\partial \pi_i^*}{\partial H_i} + \frac{\partial \pi_i^*}{\partial p_j} \frac{\partial p_j^*}{\partial H_i} + \frac{\partial \pi_i^*}{\partial p_i} \frac{\partial p_i^*}{\partial H_i} \right) \right] = 0.$$

where  $\pi_i^* = \pi(p_i^*, p_j^*, H_i^*)$ ,  $\frac{\partial \pi_i^*}{\partial x} = \frac{\partial \pi(p_i^*, p_j^*, H_i^*)}{\partial x}$  for  $x \in \{p_i, p_j, H_i\}$ ,  $\frac{\partial p_i^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_i^*, H_j^*)$ , and  $\frac{\partial p_j^*}{\partial H_i} = \frac{\partial p^*}{\partial H_i}(H_j^*, H_i^*)$ . Since  $\frac{\partial \pi_i}{\partial H_i} = \check{c} - F$ ,  $\frac{\partial \pi_i}{\partial p_j} = (p_i - \check{c}) \frac{\partial D_i}{\partial p_j}$ , and  $\frac{\partial \pi}{\partial p_i}(p_i^*, p_j^*, H_i) = 0$  by construction, this yields:

$$\mathbb{E} \left[ U'(\pi_i^*) \left( \check{c} - F + (p_i - \check{c}) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial H_i} \right) \right] = 0.$$

Dividing by  $\mathbb{E}[U'(\pi_i^*)] > 0$  yields

$$\hat{c}_i - F + (p_i - \hat{c}_i) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial H_i} = 0.$$

Observing that

$$\left( p_i - \hat{c}_i + \frac{D}{\frac{\partial D}{\partial p_i}} \right) (p_i^*, p_j^*, H_i^*) = 0$$

yields equation (15).

### B.3.3 Sign of $\frac{\partial p_j^*}{\partial H_i}$ with constant absolute risk aversion

$$\psi_1(p_i, p_j, H_i) = \psi_1^E(p_i, p_j, \hat{c}_i) - \frac{\partial D_i}{\partial p_i} \frac{\partial \hat{c}_i}{\partial p_i} < 0,$$

$$\psi_2(p_i, p_j, H_i) = \psi_2^E(p_i, p_j, \hat{c}_i) - \frac{\partial D_i}{\partial p_i} \frac{\partial \hat{c}_i}{\partial p_j} > 0,$$

and

$$\psi_3(p_i, p_j, H_i) = -\frac{\partial D_i}{\partial p_i} \frac{\partial \hat{c}_i}{\partial H_i} < 0.$$

Then:

$$\begin{aligned} \Delta &= \Delta^E - \frac{\partial D_i}{\partial p_i} \hat{\mathbb{E}} \left[ \rho(\pi_i) (\hat{c}_i - \tilde{c})^2 \right] \left( \frac{\partial D_i}{\partial p_i} \psi_1^E(p_j^*, p_i^*, \hat{c}_i) - \frac{\partial D_i}{\partial p_j} \psi_2^E(p_j^*, p_i^*, \hat{c}_i) \right) \\ &\quad - \frac{\partial D_j}{\partial p_j} \hat{\mathbb{E}} \left[ \rho(\pi_j) (\hat{c}_j - \tilde{c})^2 \right] \left( \frac{\partial D_j}{\partial p_j} \psi_1^E(p_i^*, p_j^*, \hat{c}_j) - \frac{\partial D_j}{\partial p_i} \psi_2^E(p_i^*, p_j^*, \hat{c}_j) \right) \\ &\quad + \frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j} \hat{\mathbb{E}} \left[ \rho(\pi_i) (\hat{c}_i - \tilde{c})^2 \right] \hat{\mathbb{E}} \left[ \rho(\pi_j) (\hat{c}_j - \tilde{c})^2 \right] \left( \frac{\partial D_i}{\partial p_i} \frac{\partial D_j}{\partial p_j} - \frac{\partial D_j}{\partial p_i} \frac{\partial D_i}{\partial p_j} \right) \\ &> 0 \end{aligned}$$

Thus, at the symmetric equilibrium,

$$\frac{\partial p_j^*}{\partial H_i}(H^*, H^*) = \frac{\psi_2(p^*, p^*, H^*)}{\Delta} \psi_3(p^*, p^*, H^*) < 0.$$

### B.3.4 Equilibrium price

$$\frac{\partial p_j^*}{\partial H_i}(H^*, H^*) < 0 \Leftrightarrow \hat{c}(p^*, p^*, H^*) > F.$$

Consider  $\bar{p}(c)$  defined implicitly by

$$(\bar{p}(c) - c) \frac{\partial D(\bar{p}(c), \bar{p}(c))}{\partial p_1} + D(\bar{p}(c), \bar{p}(c)) = 0$$

where  $\frac{\partial D}{\partial p_1}$  is the derivative of  $D(.,.)$  with respect to its first argument.

$$\frac{d\bar{p}}{dc} = \frac{\frac{\partial D_1}{\partial p_1}}{2 \frac{\partial D}{\partial p_1} + \frac{\partial D}{\partial p_2} + (\bar{p}(c) - c) \left( \frac{\partial^2 D}{(\partial p_1)^2} + \frac{\partial^2 D}{\partial p_2 \partial p_1} \right)} > 0$$

since  $\frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial p_2} < 0$  and  $\frac{\partial^2 D_1}{(\partial p_1)^2} + \frac{\partial^2 D_1}{\partial p_2 \partial p_1} \leq 0$  by Assumption 2. Thus,

$$\hat{c}(p^*, p^*, H^*) > F \Leftrightarrow p^* = \bar{p}(\hat{c}(p^*, p^*, H^*)) > \bar{p}(F) = p^E(F, F).$$



## C Robustness analysis: proof of Propositions 4

### C.1 Sufficient condition for the unicity of the equilibrium of the production game

We first derive a sufficient condition for  $Tr(q_1^*, q_2^*, H_1, H_2) \leq -k$ , for  $k \in (1, 2)$ . From the analysis above,

$$Tr(q_1^*, q_2^*, H_1, H_2) = \left( \frac{\partial q_1^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_1} + \frac{\partial q_2^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_2} + \frac{\partial q_1^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_1} + \frac{\partial q_2^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_2} \right) (q_1^*, q_2^*, H_1, H_2) - 2.$$

$$\text{Since } \left( \frac{\partial q_1^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_1} + \frac{\partial q_2^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_2} \right) (q_1^*, q_2^*, H_1, H_2) > 0,$$

$$\frac{\partial q_1^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_1} + \frac{\partial q_2^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_2} \leq 2 - k \Rightarrow Tr(q_1^*, q_2^*, H_1, H_2) \leq -k.$$

Since

$$\frac{\partial q_1^E}{\partial c_2} \frac{\partial \hat{c}_2}{\partial q_1} + \frac{\partial q_2^E}{\partial c_1} \frac{\partial \hat{c}_1}{\partial q_2} = - \frac{\left( P'(Q^E) + q_1^* P''(Q^E) \right) \frac{\partial \hat{c}_2}{\partial q_1} + \left( P'(Q^E) + q_2^* P''(Q^E) \right) \frac{\partial \hat{c}_1}{\partial q_2}}{P'(Q^E) \left( 3P'(Q^E) + Q^* P''(Q^E) \right)},$$

the inequality is equivalent to:

$$\begin{aligned} & - \left( P'(Q^E) + q_1^E P''(Q^E) \right) \frac{\partial \hat{c}_2}{\partial q_1} - \left( P'(Q^E) + q_2^E P''(Q^E) \right) \frac{\partial \hat{c}_1}{\partial q_2} \\ & \leq (2 - k) P'(Q^E) \left( 3P'(Q^E) + Q^E P''(Q^E) \right) \end{aligned}$$

The subsequent analysis differs depending on the concavity of  $P(\cdot)$ . Suppose first  $P''(Q) > 0$ . Then,  $-\left(P'(Q) + q_i P''(Q)\right) < -P'(Q)$ . Since  $\left| \frac{\partial \hat{c}_i}{\partial q_j} \right| \leq \left| P'(Q) \right| \bar{q}^E R$ , the left hand side is bounded above by

$$\left( -P'(Q) \right) \left( \left| \frac{\partial \hat{c}_1}{\partial q_2} \right| + \left| \frac{\partial \hat{c}_2}{\partial q_1} \right| \right) \leq 2 \left( P'(Q) \right)^2 \bar{q}^E R.$$

Since  $\frac{QP''(Q)}{(-P'(Q))} \leq 1$  by Assumption 1, the right hand side is bounded below by

$$2(2 - k) \left( P'(Q) \right)^2 \leq (2 - k) \left( P'(Q) \right)^2 \left( 3 + \frac{QP''(Q)}{P'(Q)} \right).$$

Thus,

$$\bar{q}^E R \leq (2 - k) \Rightarrow Tr(q_1^*, q_2^*, H_1, H_2) \leq -k.$$

We now derive a sufficient condition for  $Det(q_1^*, q_2^*, H_1, H_2) > 0$ . From the analysis above,

$$Det = -Tr - 1 + \left( \frac{1}{P'(Q^E) \left( 3P'(Q^E) + Q^E P''(Q^E) \right)} \left( \frac{\partial \hat{c}_1}{\partial q_1} \frac{\partial \hat{c}_2}{\partial q_2} - \frac{\partial \hat{c}_2}{\partial q_1} \frac{\partial \hat{c}_1}{\partial q_2} \right) \right).$$

Thus, since

$$\left( \frac{\partial \hat{c}_1}{\partial q_1} \frac{\partial \hat{c}_2}{\partial q_2} \right) (q_1^*, q_2^*, H_1, H_2) > 0,$$

$$\frac{\partial \hat{c}_2}{\partial q_1} \frac{\partial \hat{c}_1}{\partial q_2} < -(Tr + 1) P'(Q) \left( 3P'(Q) + Q^* P''(Q) \right) \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0.$$

The left hand side is bounded above by  $\left( P'(Q) \right)^2 (\bar{q}^E R)^2$ . The right hand side is bounded below by  $2(k-1) \left( P'(Q) \right)^2 > 0$ . Thus,

$$\bar{q}^E R \leq \sqrt{2(k-1)} \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0.$$

Thus, for any  $k \in (1, 2)$ ,

$$\bar{q}^E R \leq \min \left( \sqrt{2(k-1)}, (2-k) \right) \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0 \text{ and } Tr(q_1^*, q_2^*, H_1, H_2) \leq -k.$$

We choose  $k^* = (3 - \sqrt{3}) = \max_{k \in (1, 2)} \left( \min \left( \sqrt{2(k-1)}, (2-k) \right) \right)$ , thus, if  $P''(Q) \geq 0$ ,

$$\bar{q}^E R \leq (\sqrt{3} - 1) \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0 \text{ and } Tr(q_1^*, q_2^*, H_1, H_2) \leq -k.$$

The same derivations for  $P''(Q) < 0$  yield

$$\bar{q}^E R \leq (2\sqrt{3} - 3) \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0 \text{ and } Tr(q_1^*, q_2^*, H_1, H_2) \leq -k,$$

and for  $P''(Q) = 0$

$$\bar{q}^E R \leq 1 \Rightarrow Det(q_1^*, q_2^*, H_1, H_2) > 0 \text{ and } Tr(q_1^*, q_2^*, H_1, H_2) \leq -k,$$

## C.2 Sufficient condition for $\frac{\partial q_j^*}{\partial H_i}(H_j, H_i) \Big|_{(H^*, H^*)} < 0$

With a slight abuse, as in the main text,  $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*)$  represents  $\frac{\partial q_j^*}{\partial H_i}(H_j, H_i) \Big|_{(H^*, H^*)}$ ,

$\frac{\partial \hat{c}_i}{\partial q_i}(q^*, q^*, H^*)$  represents  $\frac{\partial \hat{c}}{\partial q_i}(q_i, q_j, H_i) \Big|_{(q^*, q^*, H^*)}$ , and  $\frac{\partial \hat{c}_i}{\partial q_j}(q^*, q^*, H^*)$  repre-

sents  $\frac{\partial \widehat{c}}{\partial q_j}(q_i, q_j, H_j) \Big|_{(q^*, q^*, H^*)}$ .

**Lemma 4** Consider a symmetric equilibrium,  $H_i = H_j = H^*$ , and  $q_i = q_j = q^*(H^*, H^*)$ .

$$\left( P'(Q) - \frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial \widehat{c}_i}{\partial q_j} \right) (q^*, q^*, H^*) < 0 \Leftrightarrow \frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0.$$

**Proof.** We first prove that

$$\left( P'(Q) - \frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial \widehat{c}_i}{\partial q_j} \right) (q^*, q^*, H^*) < 0 \Rightarrow \frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0.$$

$$\begin{aligned} \Delta &= \psi_1(q_i^*, q_j^*, H_i) \psi_1(q_j^*, q_i^*, H_j) - \psi_2(q_i^*, q_j^*, H_i) \psi_2(q_j^*, q_i^*, H_j) \\ &= \left( \begin{aligned} &\left( 2P'(Q) + q_i P''(Q) - \frac{\partial \widehat{c}_i}{\partial q_i} \right) \left( 2P'(Q) + q_j P''(Q) - \frac{\partial \widehat{c}_j}{\partial q_j} \right) \\ &- \left( P'(Q) + q_i P''(Q) - \frac{\partial \widehat{c}_i}{\partial q_j} \right) \left( P'(Q) + q_j P''(Q) - \frac{\partial \widehat{c}_j}{\partial q_i} \right) \end{aligned} \right) (q_i^*, q_j^*, H_i, H_j) \\ &= \left( \begin{aligned} &-P'(Q) \left( \frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial \widehat{c}_j}{\partial q_j} - P'(Q) \right) \\ &+ \left( P'(Q) - \frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial \widehat{c}_i}{\partial q_j} \right) \left( P'(Q) + q_j P''(Q) \right) \\ &+ \left( P'(Q) - \frac{\partial \widehat{c}_j}{\partial q_j} + \frac{\partial \widehat{c}_i}{\partial q_j} \right) \left( P'(Q) + q_i P''(Q) \right) \end{aligned} \right) (q_i^*, q_j^*, H_i, H_j) \end{aligned}$$

Thus,

$$\left( P'(Q) - \frac{\partial \widehat{c}}{\partial q_i} + \frac{\partial \widehat{c}}{\partial q_j} \right) (q_i^*, q_j^*, H_i, H_j) < 0 \text{ for } i = 1, 2 \Rightarrow \Delta > 0.$$

Thus, since  $\psi_3(q_i, q_j, H_i) > 0$ ,

$$\psi_2(q^*, q^*, H^*) = \left( P'(Q) + q_i P''(Q) - \frac{\partial \widehat{c}_i}{\partial q_j} \right) (q^*, q^*, H^*) < 0 \Leftrightarrow \frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0.$$

To complete the proof, we show that  $\psi_2(q^*, q^*, H^*) < 0$ . Suppose  $\psi_2(q^*, q^*, H^*) > 0$ , then

$$\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) > 0 \Leftrightarrow H^* < q^* \Leftrightarrow \frac{\partial \widehat{c}_i}{\partial q_j} > 0 \Rightarrow \psi_2(q^*, q^*, H^*) < 0$$

which is a contradiction.

Reciprocally,

$$\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0 \Leftrightarrow H^* > q^* \Leftrightarrow \frac{\partial \widehat{c}_i}{\partial q_j} < 0 \Rightarrow \left( P'(Q) - \frac{\partial \widehat{c}_i}{\partial q_i} + \frac{\partial \widehat{c}_i}{\partial q_j} \right) (q^*, q^*, H^*) < 0$$

■

Since  $\frac{\partial \hat{c}_i}{\partial q_i}(q^*, q^*, H^*) > 0$  and  $|P'(Q^*)| < 0$ ,

$$\left| \frac{\partial \hat{c}_i}{\partial q_j}(q^*, q^*, H^*) \right| \leq |P'(Q^*)| \Rightarrow \left( P'(Q) - \frac{\partial \hat{c}_i}{\partial q_i} + \frac{\partial \hat{c}_i}{\partial q_j} \right)(q^*, q^*, H^*) < 0$$

Thus, from the previous Lemma,

$$\left| \frac{\partial \hat{c}_i}{\partial q_j} \right| \leq |P'(Q)| \Rightarrow \frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$$

$\Rightarrow$

$$\bar{q}^E R \leq 1 \Rightarrow \frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0.$$

### C.3 Global sufficient condition for quantity competition

Since  $(\sqrt{3} - 1) < 1$  and  $(2\sqrt{3} - 3) < 1$ , the sufficient condition for unicity of the equilibrium and  $\frac{\partial q_j^*}{\partial H_i}(H^*, H^*) < 0$  is  $\bar{q}^E R \leq (\sqrt{3} - 1)$  if  $P''(Q) > 0$ ,  $\bar{q}^E R \leq (2\sqrt{3} - 3)$  if  $P''(Q) < 0$ , and  $\bar{q}^E R \leq 1$  if  $P''(Q) = 1$ .

## D Strategic incentives to commit (Proposition 5)

### D.1 Comparing $\Omega(C, NC)$ and $\Omega(NC, NC)$ if firms compete in price

Suppose firm 2 plays  $NC$ , while firm 1 plays  $C$ . At  $t = 2$ , firm 2 chooses  $\bar{H}_2 = D(p_2, p_1)$ . At  $t = 1$ , firms simultaneously select prices  $(\bar{p}_1(H_1), \bar{p}_2(H_1))$  that solve:

$$\begin{cases} D(\bar{p}_1, \bar{p}_2) + (\bar{p}_1 - \hat{c}(\bar{p}_1, \bar{p}_2, H_1)) \frac{\partial D}{\partial p_1}(\bar{p}_1, \bar{p}_2) = 0 \\ D(\bar{p}_2, \bar{p}_1) + (\bar{p}_2 - F) \frac{\partial D}{\partial p_2}(\bar{p}_2, \bar{p}_1) = 0 \end{cases}$$

At  $t = 0$ , firm 1 selects  $\bar{H}_1$  that maximizes  $Z(H_1) = \mathbb{E}[U(\pi(\bar{p}_1(H_1), \bar{p}_2(H_1), H_1))]$ .

As in the Cournot case, if firm 1 chooses  $H_1 = D(p^E(F, F), p^E(F, F))$ ,  $p_1 = p_2 = p^E(F, F)$  is a solution of the system, hence the unique equilibrium.

Both firms receive  $\Omega(NC, NC)$ . Thus,  $\Omega(C, NC) \geq \Omega(NC, NC)$ .

Then

$$\frac{dZ}{dH_1}(D(p^E(F, F), p^E(F, F))) = \frac{\partial D_1}{\partial p_2}(p^E(F, F) - F) \frac{\partial \bar{p}_2}{\partial H_1} \mathbb{E}[U'(\pi_1)] < 0.$$

Thus, if the firm hedges  $(\frac{1}{2} - \varepsilon)$  where  $\varepsilon > 0$  is arbitrarily small, by continuity,  $Z(\frac{1}{2} - \varepsilon) > Z(\frac{1}{2})$ . Thus,  $\Omega(C, NC) > \Omega(NC, NC)$ .

## D.2 Comparing $\Omega(C, C)$ and $\Omega(NC, NC)$ if firms compete in quantity

$$\Omega(C, C) = \mathbb{E} \left[ U \left( (P(Q^*) - F) \frac{Q^*}{2} + (H^* - Q^*)(\omega - F) \right) \right] < U \left( (P(Q^*) - F) \frac{Q^*}{2} \right)$$

since  $U(\cdot)$  is concave, and

$$\Omega(NC, NC) = U \left( (P(2q^E(F, F)) - F) q^E(F, F) \right).$$

For  $x \geq 0$ , denote

$$f(x) = (P(2x) - F)x.$$

Condition 1 implies that  $f(\cdot)$  is globally concave and admits a unique maximum  $x^*$  defined by:

$$f'(x^*) = P(2x^*) - F + 2x^*P'(2x^*) = 0.$$

Then,

$$\begin{aligned} f'(q^E(F, F)) &= \left( -q^E(F, F)P'(2q^E(F, F)) + 2q^E(F, F)P'(2q^E(F, F)) \right) \\ &= q^E(F, F)P'(2q^E(F, F)) \\ &< 0, \end{aligned}$$

hence  $q^E(F, F) > x^*$ . Then,  $f(q^E(F, F)) > f(q^*)$  since  $q^* > q^E(F, F)$  and  $f(\cdot)$  is decreasing for  $x \geq x^*$ . Thus:

$$\Omega(C, C) < U(f(q^*)) < U(f(q^E(F, F))) = \Omega(NC, NC).$$

## D.3 Hotelling competition

### D.3.1 Comparing $\Omega(C, C)$ and $\Omega(NC, NC)$

Under the conditions of Proposition 5

$$\begin{aligned} \hat{c}(p_i, p_j, H_i) &= F + \rho\sigma^2(D_i(p_i, p_j) - H_i) \\ &= F + \rho\sigma^2 \left( \frac{p_j - p_i}{2t} + \frac{1}{2} - H_i \right) \end{aligned}$$

Thus, for  $i = 1, 2$ , the first-order conditions (14) are

$$D(p_i, p_j) + \frac{\partial D}{\partial p_i}(p_i, p_j)(p_i - (F + \rho\sigma^2(D(p_i, p_j) - H_i))) = 0$$

$$\begin{aligned}
&\Leftrightarrow \frac{1}{2} + \frac{p_j - p_i}{2t} - \frac{1}{2t} \left( p_i - \left( F + \rho\sigma^2 \left( \frac{p_j - p_i}{2t} + \frac{1}{2} - H_i \right) \right) \right) = 0 \\
&\Leftrightarrow \left( 2 + \frac{\rho\sigma^2}{2t} \right) p_i - \left( 1 + \frac{\rho\sigma^2}{2t} \right) p_j = t + F + \rho\sigma^2 \left( \frac{1}{2} - H_i \right)
\end{aligned}$$

which yield equilibrium prices for  $i = 1, 2$

$$\begin{aligned}
p^*(H_i, H_j) &= t + F + \frac{\rho\sigma^2}{2} \left( 1 - \frac{(4t + \rho\sigma^2) H_i + (2t + \rho\sigma^2) H_j}{3t + \rho\sigma^2} \right) \\
&= F + \left( 1 + \frac{1}{2a} \left( 1 - \frac{(1 + 4a) H_i + (1 + 2a) H_j}{1 + 3a} \right) \right) t
\end{aligned}$$

where  $a = \frac{t}{\rho\sigma^2}$ . Thus:

$$\frac{\partial p_j^*}{\partial H_i} = \frac{\partial p^*(H_j, H_i)}{\partial H_i} = -\frac{1 + 2a}{2a(1 + 3a)} t < 0.$$

The first-order conditions (15) become

$$\left( \rho\sigma^2 (D(p_i^*, p_j^*) - H_i) + D(p_i^*, p_j^*) \frac{\partial p_j^*}{\partial H_i} \right) (H_i^*, H_j^*) = 0$$

since  $\widehat{c}_i = F + \rho\sigma^2 (D(p_i, p_j) - H_i)$  and  $\frac{\partial D}{\partial p_j}(p_i, p_j) = -\frac{\partial D}{\partial p_i}(p_i, p_j) = \frac{1}{2t}$ .

Consider now a symmetric equilibrium:  $H_i^* = H_j^* = H^*$ , hence  $p_i^* = p_j^* = p^*$ . Since the equilibrium is symmetric,  $D_i(p_i^*, p_j^*) = \frac{1}{2}$ . Thus, the first-order conditions (15) further simplify to:

$$\frac{1}{2} - H^* = \frac{1 + 2a}{4(1 + 3a)} > 0.$$

At the symmetric equilibrium, Hotelling equilibrium price is:

$$p^* = t + \widehat{c}(p^*, p^*, H^*) = t + F + \rho\sigma^2 \left( \frac{1}{2} - H^* \right)$$

$$\begin{aligned}
&\Leftrightarrow \\
&p^* - F = \left( 1 + \frac{1 + 2a}{4a(1 + 3a)} \right) t
\end{aligned}$$

Thus,

$$\begin{aligned}\Omega(C, C) &= \frac{1}{2} \left( 1 + \frac{1+2a}{4a(1+3a)} \right) t - \frac{\rho\sigma^2}{2} \left( \frac{1+2a}{4a(1+3a)} \right)^2 \\ &= \left( 1 + \frac{(1+2a)(3+10a)}{16a(1+3a)^2} \right) \frac{t}{2}.\end{aligned}$$

If both firms play Not Commit, they fully cover their exposure at  $t = 3$ . Knowing this, they play at  $t = 2$  symmetric Hotelling game with constant marginal cost equal to the forward price. Thus,

$$\Omega(C, C) = \frac{t}{2}$$

and

$$\Omega(C, C) - \Omega(NC, NC) = \frac{(1+2a)(3+10a)t}{16a(1+3a)^2} \frac{t}{2} > 0.$$

### D.3.2 Comparing $\Omega(C, C)$ and $\Omega(NC, C)$

Suppose again firm 2 plays  $NC$ , while firm 1 plays  $C$ . We prove that  $\Omega(NC, C) < \Omega(C, C)$ . The equilibrium prices  $(\bar{p}_1(H_1), \bar{p}_2(H_1))$  are given by the Hotelling formula:

$$\begin{cases} \bar{p}_1 = t + \frac{1}{3} \left( 2 \left( F + \rho\sigma^2 \left( \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 \right) \right) + F \right) = t + F + \frac{2t}{3a} \left( \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 \right) \\ \bar{p}_2 = t + \frac{1}{3} \left( 2F + \left( F + \rho\sigma^2 \left( \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 \right) \right) \right) = t + F + \frac{t}{3a} \left( \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 \right) \end{cases}.$$

Then,

$$\bar{p}_2 - \bar{p}_1 = -\frac{t}{3a} \left( \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 \right) = -2t \frac{1}{1+6a} \left( \frac{1}{2} - H_1 \right)$$

and

$$D_1 - H_1 = \frac{\bar{p}_2 - \bar{p}_1}{2t} + \frac{1}{2} - H_1 = \frac{6a}{1+6a} \left( \frac{1}{2} - H_1 \right).$$

Thus,

$$\begin{cases} \bar{p}_1 - F = \left( 1 + \frac{4}{1+6a} \left( \frac{1}{2} - H_1 \right) \right) t \\ \bar{p}_2 - F = \left( 1 + \frac{2}{1+6a} \left( \frac{1}{2} - H_1 \right) \right) t \end{cases},$$

and

$$\frac{\partial \bar{p}_2}{\partial H_1} = -\frac{2t}{1+6a} < 0.$$

Maximization over  $H_1$  yields:

$$\frac{1}{2t} (\bar{p}_1 - F + \rho\sigma^2 (H_1 - D_1)) \frac{\partial \bar{p}_2}{\partial H_1} - \rho\sigma^2 (H_1 - D_1) = 0$$

$\Leftrightarrow$

$$- \left( \left( 1 + \frac{4}{1+6a} \left( \frac{1}{2} - \bar{H}_1 \right) \right) t - \rho\sigma^2 \frac{6a}{1+6a} \left( \frac{1}{2} - \bar{H}_1 \right) \right) \frac{2t}{1+6a} + 2t\rho\sigma^2 \frac{6a}{1+6a} \left( \frac{1}{2} - \bar{H}_1 \right) = 0$$

$\Leftrightarrow$

$$\frac{1}{2} - \bar{H}_1 = \frac{1+6a}{4(2+9a)}.$$

Thus,

$$\bar{p}_2 - F = \left( 1 + \frac{1}{2(2+9a)} \right) t,$$

and

$$D(\bar{p}_2, \bar{p}_1) = \frac{1}{2} \left( 1 + \frac{1}{2(2+9a)} \right),$$

and

$$\Omega(NC, C) = \left( 1 + \frac{1}{2(2+9a)} \right)^2 \frac{t}{2}.$$

Thus

$$\Omega(C, C) > \Omega(NC, C) \iff 1 + \frac{(1+2a)(3+10a)}{16a(1+3a)^2} > 1 + \frac{1}{2+9a} + \frac{1}{4(2+9a)^2}$$

$\iff$

$$\frac{(1+2a)(3+10a)(2+9a)^2}{36a(1+4a)(1+3a)^2} > 1$$

which is verified numerically for all  $a \geq 0$ .



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